Static Program Analysis

Yue Li and Tian Tan

2020 Spring

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Pointer Analysis
Context Sensitivity

Nanjing University

Tian Tan

2020

Problem of Context-Insensitive Pointer Analysis

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void main() {
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Constant propagation: i = ?

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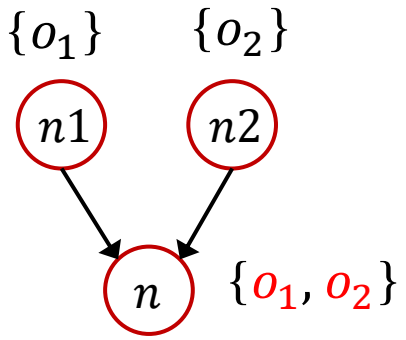

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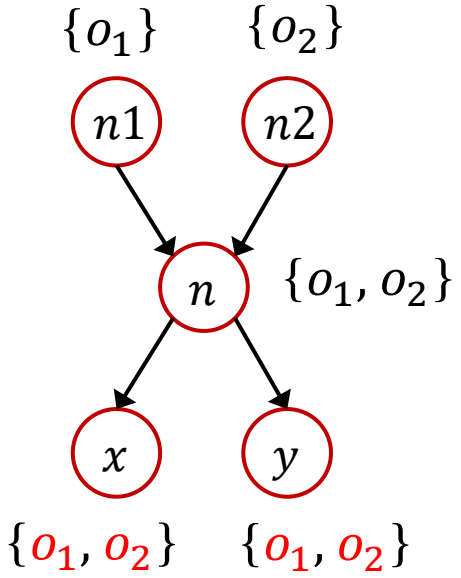

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Context insensitivity, `x.get()`:

- 2 call targets

Constant propagation

- `i` = NAC

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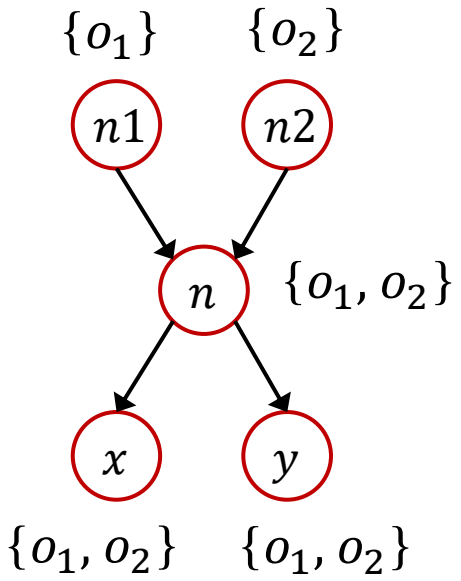

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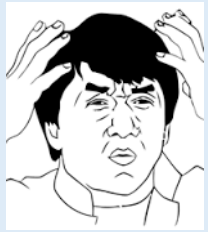

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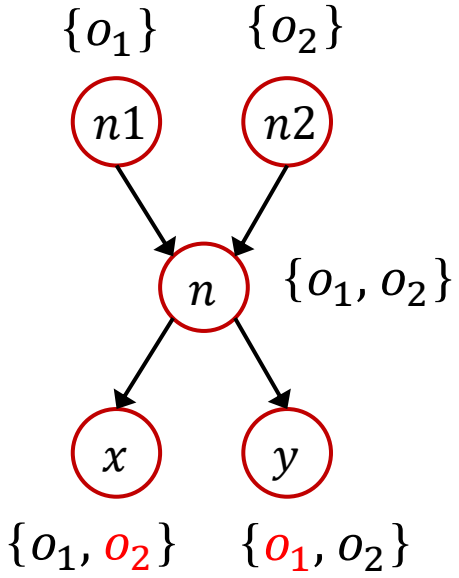

Via Context-Sensitive Pointer Analysis

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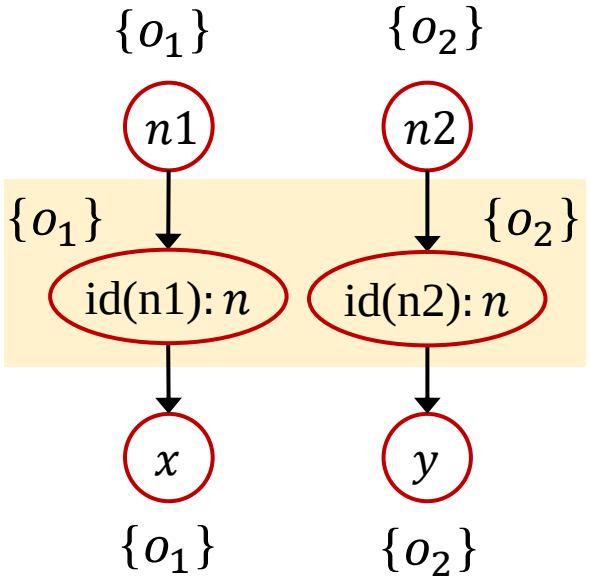

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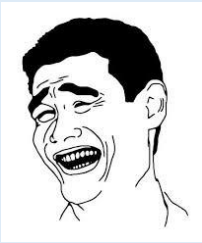
Context sensitivity, `x.get()`:

- 1 call targets
- 0 false positive

Constant propagation

- `i` = 1



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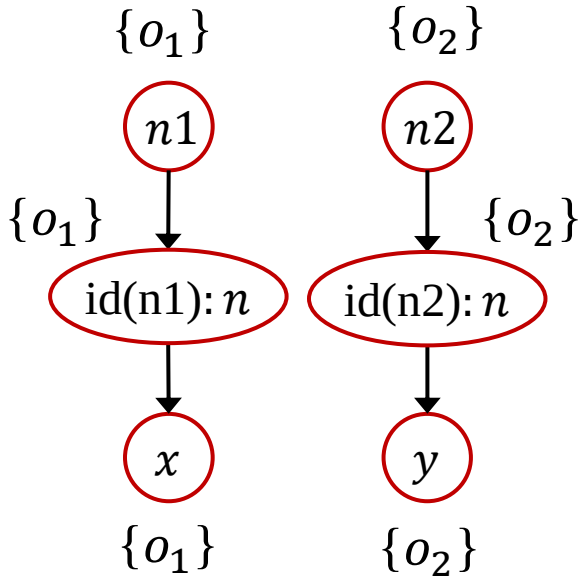
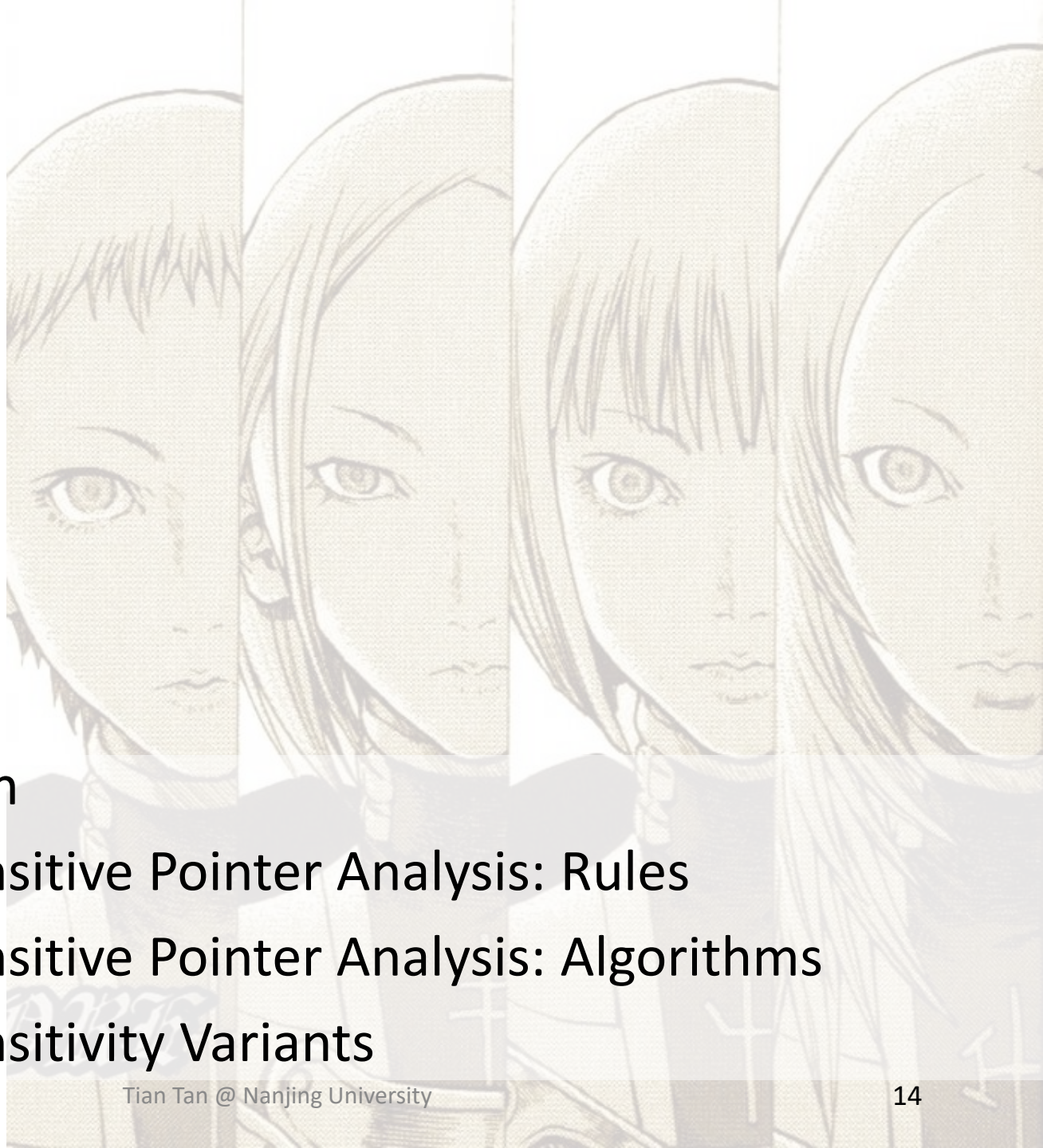


Contents



1. Introduction
2. Context Sensitive Pointer Analysis: Rules
3. Context Sensitive Pointer Analysis: Algorithms
4. Context Sensitivity Variants

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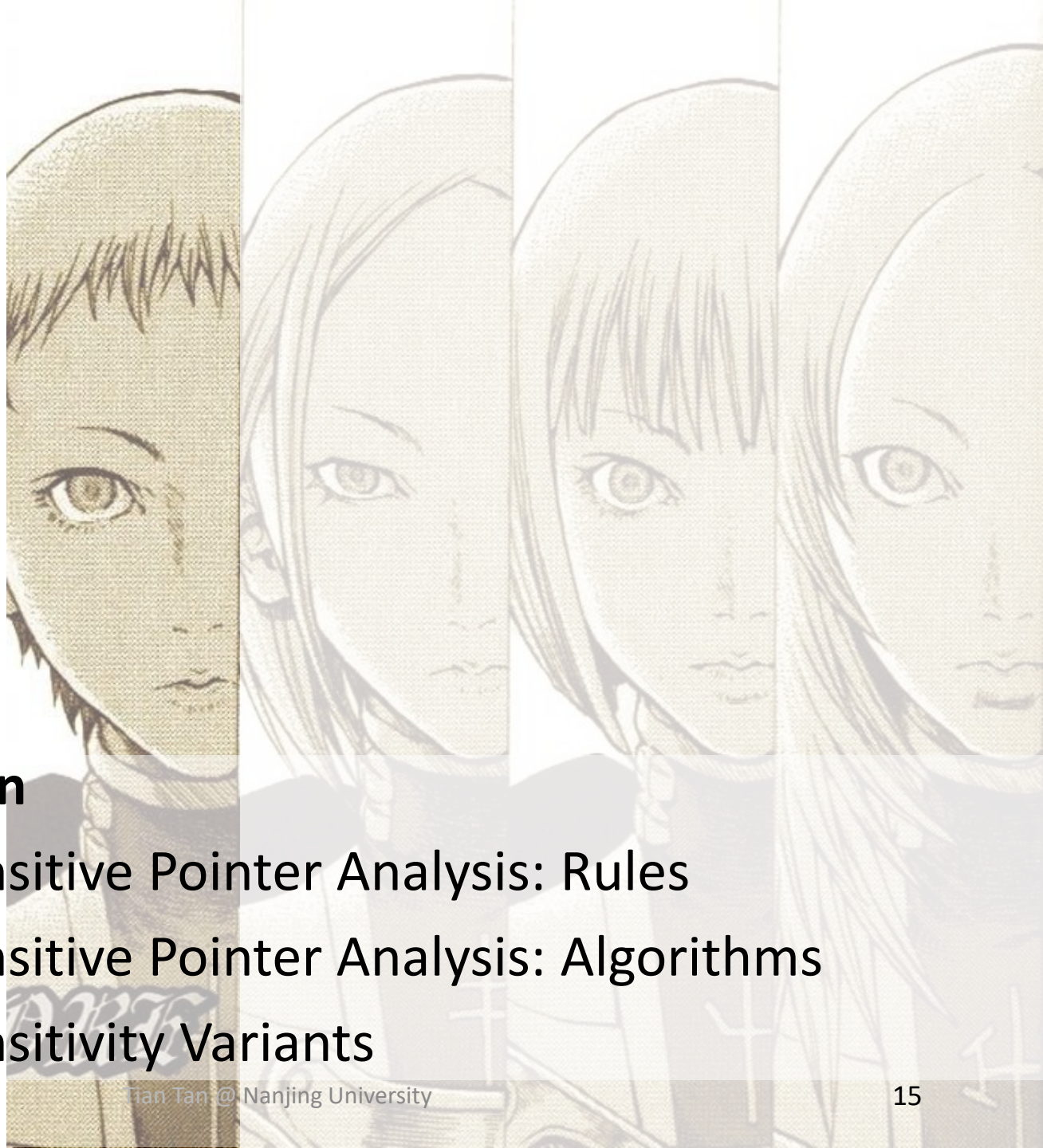

1. **Introduction**
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Imprecision of Context Insensitivity (C.I.)

- In dynamic execution, a method may be called multiple times under different calling contexts

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- Under different calling contexts, the variables of the method may point to different objects

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Imprecision of Context Insensitivity (C.I.)

- In dynamic execution, a method may be called multiple times under different calling contexts
- Under different calling contexts, the variables of the method may point to different objects
- In C.I. pointer analysis, objects under different contexts are mixed and propagated to other parts of program (through return values or side-effects), causing spurious data flows

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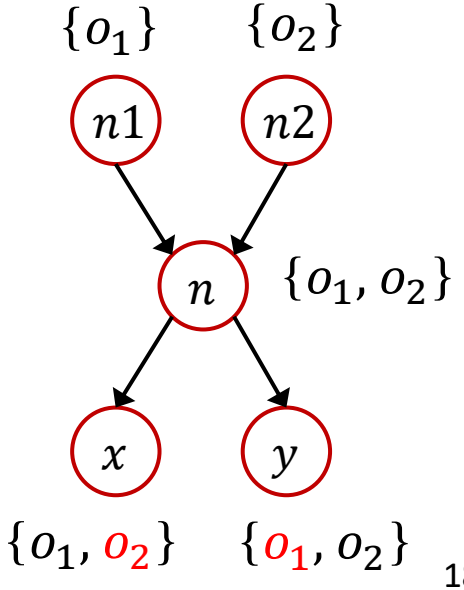

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  - Which represents each context of a method as a chain of call sites, i.e.,
    a call site of the method,
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  (abstract call stacks in dynamic execution)

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In call-site sensitivity, what are the contexts for method `id(Number)`?

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You will see other variants of context sensitivity in the next lecture

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Cloning-Based Context Sensitivity

The most straightforward approach to implement context sensitivity

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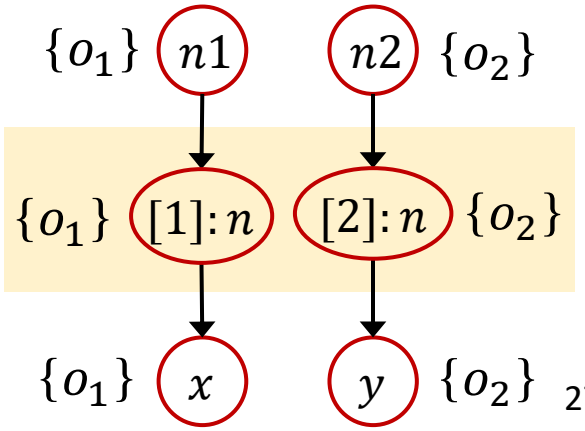

Context-Sensitive Heap

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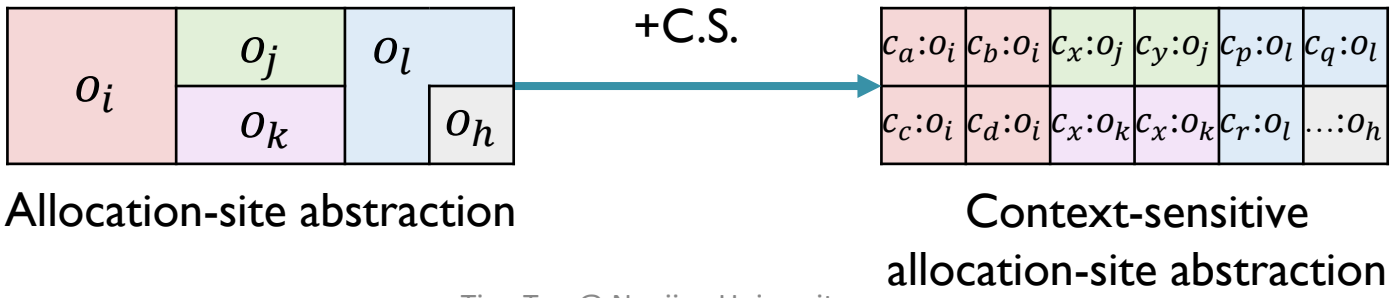
  - Context-sensitive heap abstraction provides a finer-grained heap model over allocation-site abstraction



Why Context-Sensitive Heap Improve Precision?

- In dynamic execution, an allocation site can create multiple objects under different calling contexts

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X newX(…) {
  X x = new X();
  …
  return x;
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➢ In pointer analysis, analyzing such code without heap contexts may lose precision by merging the data flows of different contexts to one abstract object

➢ In contrast, distinguishing different objects from the same allocation site by heap contexts gains precision

Context-Sensitive Heap: Example

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5  n = x1.f;
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C.S., no C.S. heap

Variable	Object
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3:x	o_8
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Field	**Object**
o_8.f	o_1

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Field	**Object**
o_8.f	o_1, o_2

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x2	o_8
n	?

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o_8.f	o_1, o_2

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10    return x;
11  }
12  class X {
13     Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

Spurious data flow
due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13     Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2

Field	Object
o_8.f	o_1, o_2

Spurious data flow due to lack of C.S. heap

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1

?

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3:o_8

Object with heap context

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2

Field	Object
o_8.f	o_1, o_2

Spurious data flow due to lack of C.S. heap

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3:o_8

?

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	$3{:}o_8$
x1	$3{:}o_8$
Field	**Object**
$3{:}o_8$.f	o_1

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	$3{:}o_8$
x1	$3{:}o_8$
4:p	o_2
4:x	$4{:}o_8$
Field	**Object**
$3{:}o_8$.f	o_1

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	$3{:}o_8$
x1	$3{:}o_8$
4:p	o_2
4:x	$4{:}o_8$
x2	$4{:}o_8$
Field	**Object**
$3{:}o_8$.f	o_1
$4{:}o_8$.f	o_2

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	$3{:}o_8$
x1	$3{:}o_8$
4:p	o_2
4:x	$4{:}o_8$
x2	$4{:}o_8$
n	?
Field	**Object**
$3{:}o_8$.f	o_1
$4{:}o_8$.f	o_2

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3:o_8
x1	3:o_8
4:p	o_2
4:x	4:o_8
x2	4:o_8
n	o_1
Field	**Object**
3:o_8.f	o_1
4:o_8.f	o_2

Spurious data flow due to lack of C.S. heap

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13     Number f;
14  }
```

C.S., no C.S. heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	o_8
x1	o_8
4:p	o_2
4:x	o_8
x2	o_8
n	o_1, o_2
Field	**Object**
o_8.f	o_1, o_2

Spurious data flow due to lack of C.S. heap

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	3:o_8
x1	3:o_8
4:p	o_2
4:x	4:o_8
x2	4:o_8
n	o_1
Field	**Object**
3:o_8.f	o_1
4:o_8.f	o_2

Context-sensitive heap improves precision

Context-Sensitive Heap: Example

```
1   n1 = new One();
2   n2 = new Two();
3   x1 = newX(n1);
4   x2 = newX(n2);
5   n = x1.f;
6
7   X newX(Number p) {
8     X x = new X();
9     x.f = p;
10    return x;
11  }
12  class X {
13    Number f;
14  }
```

C.I. + C.S. heap

Variable	Object
n1	o_1
n2	o_2
p	o_1, o_2
x	$3{:}o_8, 4{:}o_8$
x1	$3{:}o_8, 4{:}o_8$
x2	$3{:}o_8, 4{:}o_8$
n	o_1, o_2
Field	**Object**
$3{:}o_8.f$	o_1, o_2
$4{:}o_8.f$	o_1, o_2

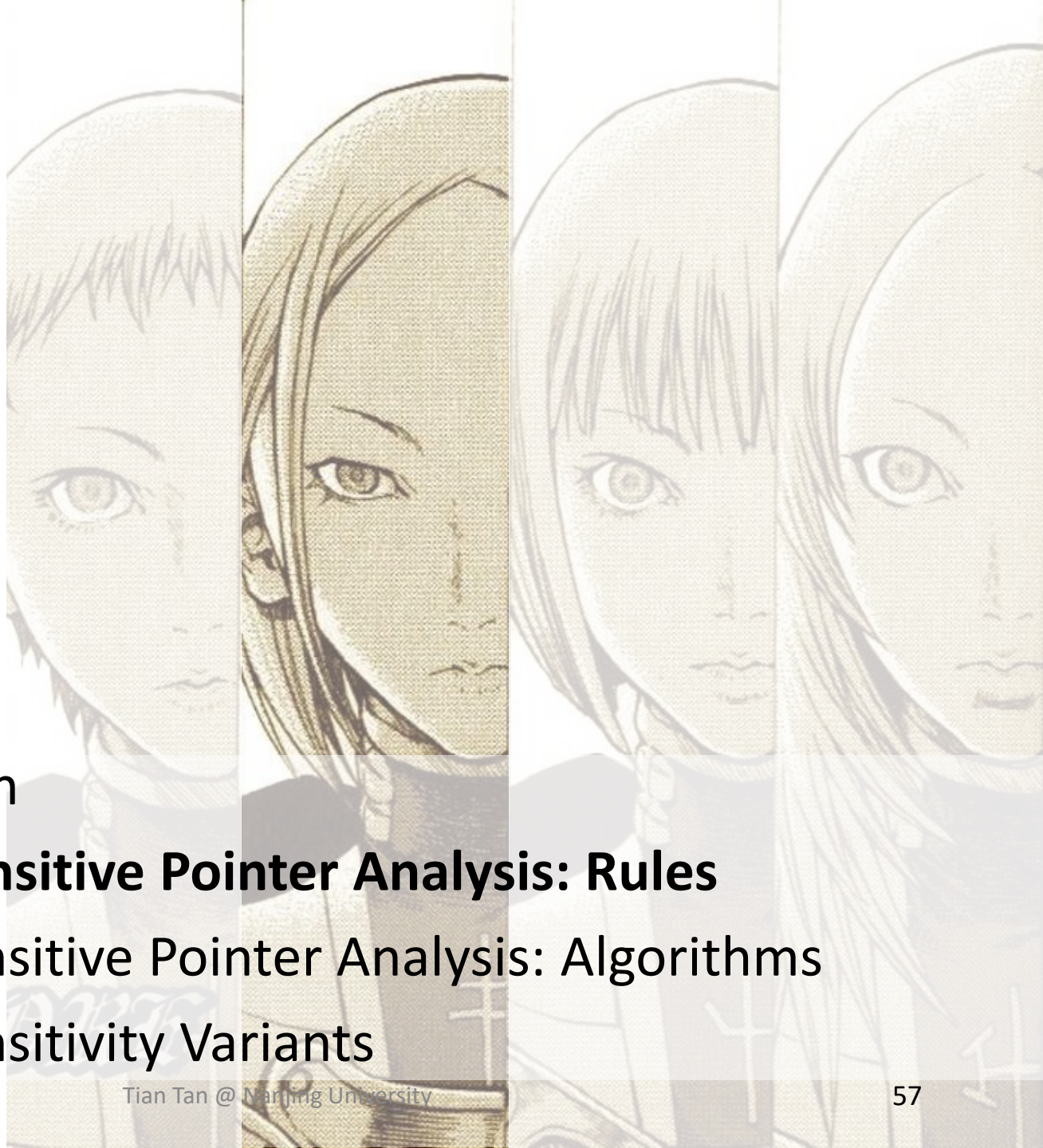
Without C.S., C.S. heap cannot improve precision

C.S. + C.S heap

Variable	Object
n1	o_1
n2	o_2
3:p	o_1
3:x	$3{:}o_8$
x1	$3{:}o_8$
4:p	o_2
4:x	$4{:}o_8$
x2	$4{:}o_8$
n	o_1
Field	**Object**
$3{:}o_8.f$	o_1
$4{:}o_8.f$	o_2

Context-sensitive heap improves precision

Contents



1. Introduction
2. **Context Sensitive Pointer Analysis: Rules**
3. Context Sensitive Pointer Analysis: Algorithms
4. Context Sensitivity Variants

Domains and Notations

In context-sensitive analysis,
program elements are qualified by **contexts**

Context:	$c, c', c'' \in \mathrm{C}$
Context-sensitive methods:	$c\colon m \in \mathrm{C} \times \mathrm{M}$
Context-sensitive variables:	$c\colon x, c'\colon y \in \mathrm{C} \times \mathrm{V}$
Context-sensitive objects:	$c\colon o_i, c'\colon o_j \in \mathrm{C} \times \mathrm{O}$
Fields:	$f, g \in \mathrm{F}$
Instance fields:	$c\colon o_i.f, c'\colon o_j.g \in \mathrm{C} \times \mathrm{O} \times \mathrm{F}$
Context-sensitive pointers:	$\mathrm{CSPointer} = (\mathrm{C} \times \mathrm{V}) \cup (\mathrm{C} \times \mathrm{O} \times \mathrm{F})$
Points-to relations:	$pt : \mathrm{CSPointer} \rightarrow \mathcal{P}(\mathrm{C} \times \mathrm{O})$

Rules

Kind	Statement	Rule (under context c)
New	`i: x = new T()`	$\dfrac{}{c: o_i \in pt(c: x)}$
Assign	`x = y`	$\dfrac{c': o_i \in pt(c: y)}{c': o_i \in pt(c: x)}$
Store	`x.f = y`	$\dfrac{c': o_i \in pt(c: x), c'': o_j \in pt(c: y)}{c'': o_j \in pt(c': o_i.f)}$
Load	`y = x.f`	$\dfrac{c': o_i \in pt(c: x), c'': o_j \in pt(c': o_i.f)}{c'': o_j \in pt(c: y)}$

Rules

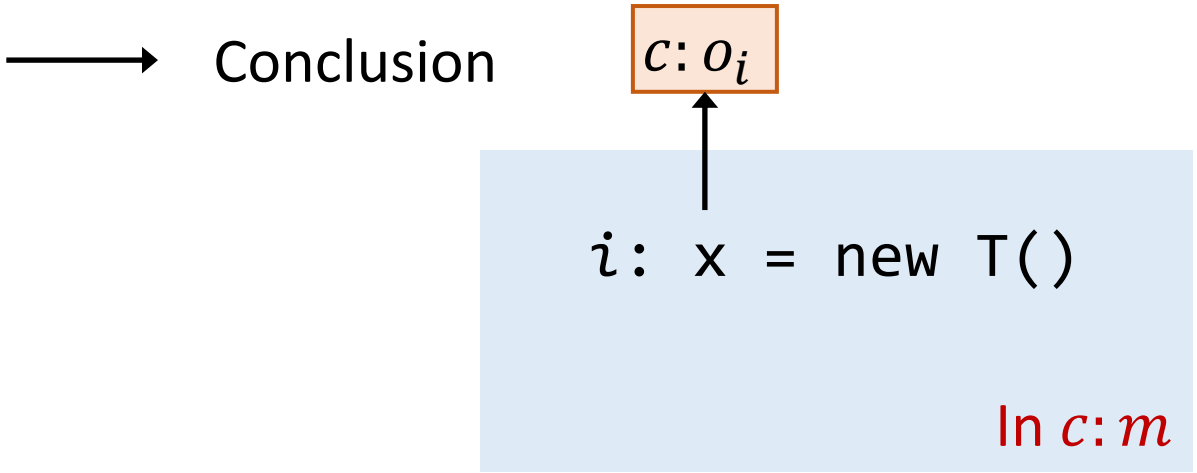
Kind	Statement	Rule
New	`i: x = new T()`	$\dfrac{}{o_i \in pt(x)}$
Assign	`x = y`	$\dfrac{o_i \in pt(y)}{o_i \in pt(x)}$
Store	`x.f = y`	$\dfrac{o_i \in pt(x), o_j \in pt(y)}{o_j \in pt(o_i.f)}$
Load	`y = x.f`	$\dfrac{o_i \in pt(x), o_j \in pt(o_i.f)}{o_j \in pt(y)}$

Rules

Kind	Statement	Rule (under context c)
New	`i: x = new T()`	$\dfrac{}{c: o_i \in pt(c: x)}$
Assign	`x = y`	$\dfrac{c': o_i \in pt(c: y)}{c': o_i \in pt(c: x)}$
Store	`x.f = y`	$\dfrac{c': o_i \in pt(c: x), c'': o_j \in pt(c: y)}{c'': o_j \in pt(c': o_i.f)}$
Load	`y = x.f`	$\dfrac{c': o_i \in pt(c: x), c'': o_j \in pt(c': o_i.f)}{c'': o_j \in pt(c: y)}$

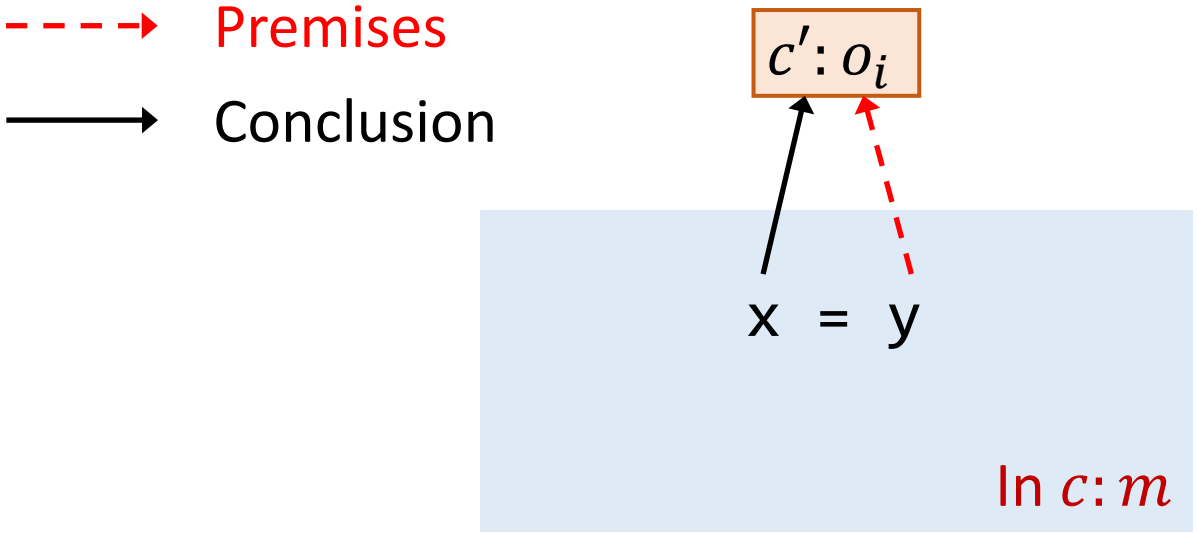
Rule: New

$$\frac{}{c{:}\,o_i \in pt(c{:}\,x)}$$



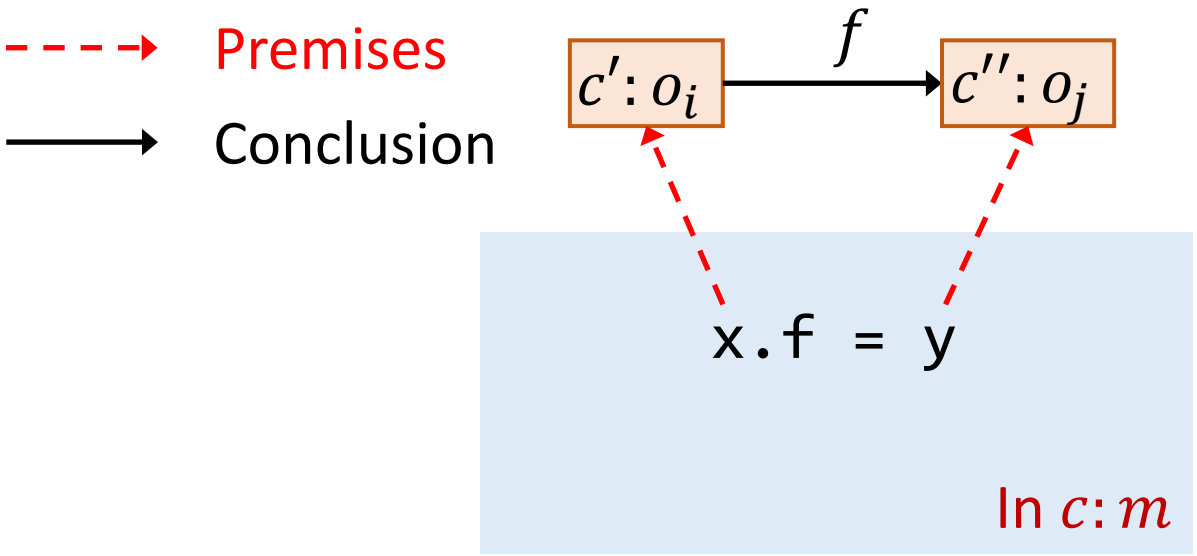
Rule: Assign

$$\frac{c' : o_i \in pt(c : y)}{c' : o_i \in pt(c : x)}$$



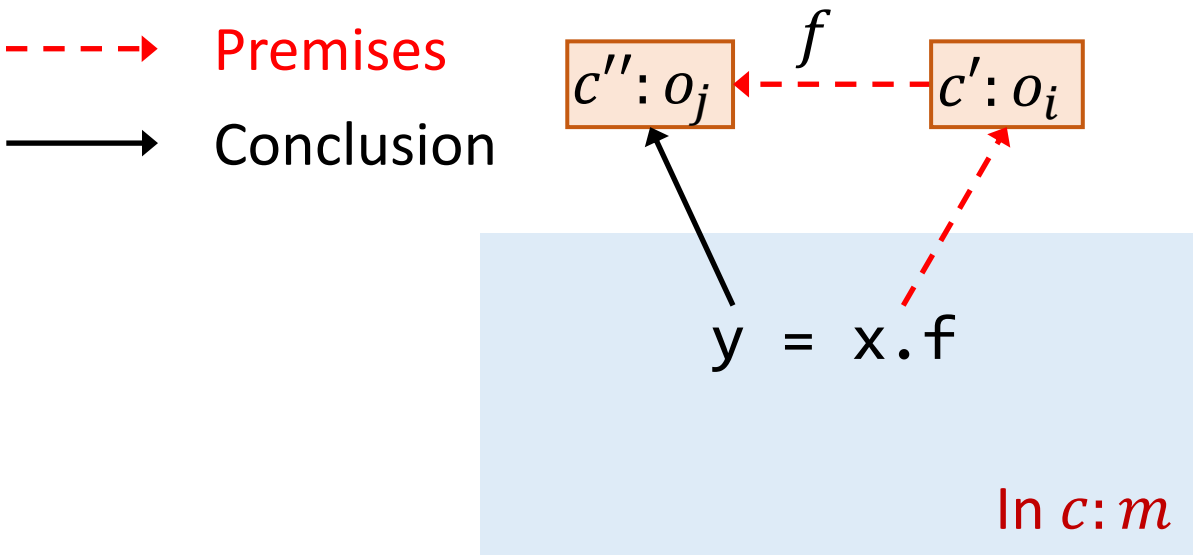
Rule: Store

$$\frac{c' : o_i \in pt(c : x), c'' : o_j \in pt(c : y)}{c'' : o_j \in pt(c' : o_i . f)}$$



Rule: Load

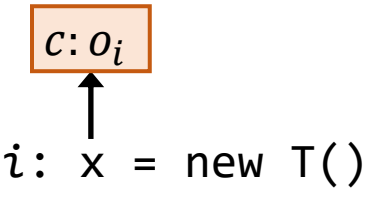
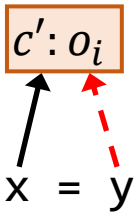
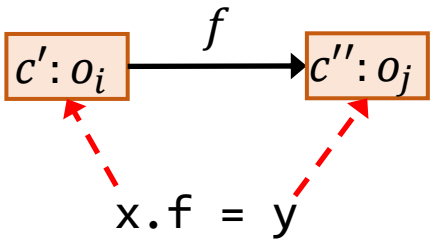
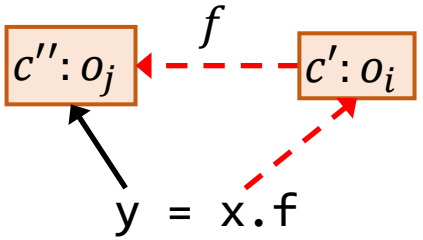
$$\frac{c' : o_i \in pt(c : x), c'' : o_j \in pt(c' : o_i . f)}{c'' : o_j \in pt(c : y)}$$



Rules

Premises

Conclusion

Kind	Rule	Illustration
New	$\dfrac{}{c{:}\,o_i \in pt(c{:}\,x)}$	
Assign	$\dfrac{c'{:}\,o_i \in pt(c{:}\,y)}{c'{:}\,o_i \in pt(c{:}\,x)}$	
Store	$\dfrac{c'{:}\,o_i \in pt(c{:}\,x), c''{:}\,o_j \in pt(c{:}\,y)}{c''{:}\,o_j \in pt(c'{:}\,o_i.f)}$	
Load	$\dfrac{c'{:}\,o_i \in pt(c{:}\,x), c''{:}\,o_j \in pt(c'{:}\,o_i.f)}{c''{:}\,o_j \in pt(c{:}\,y)}$	

Rule: Call

Caller context: c

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c: x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(c: aj), 1 \le j \le n \\ c''': o_v \in pt(c^t: m_{ret}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t: m_{this}) \\ c'': o_u \in pt\left(c^t: m_{pj}\right), 1 \le j \le n \\ c''': o_v \in pt(c: r) \end{array}}$

Rule: Call

Caller context: c

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c: x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(c: aj), 1 \le j \le n \\ c''': o_v \in pt(c^t: m_{ret}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t: m_{this}) \\ c'': o_u \in pt(c^t: m_{pj}), 1 \le j \le n \\ c''': o_v \in pt(c: r) \end{array}}$

- Dispatch(o_i, k): resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)

Rule: Call

Caller context: c
Callee context: c^t

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c{:}\,x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(c{:}\,aj), 1 \le j \le n \\ c''': o_v \in pt(c^t{:}\,m_{ret}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t{:}\,m_{this}) \\ c'': o_u \in pt(c^t{:}\,m_{pj}), 1 \le j \le n \\ c''': o_v \in pt(c{:}\,r) \end{array}}$

- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- ➢ $\text{Select}(c, l, c': o_i)$: selects context for target method m, based on the information available at call site l

Rule: Call

Caller context: c
Callee context: c^t

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c{:}\,x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(c{:}\,aj), 1 \le j \le n \\ c''': o_v \in pt(c^t{:}\,m_{ret}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t{:}\,m_{this}) \\ c'': o_u \in pt(c^t{:}\,m_{pj}), 1 \le j \le n \\ c''': o_v \in pt(c{:}\,r) \end{array}}$

Caller context: c

```
1 …
2 x = id(n1);
3 y = id(n2);
4 …
```

Select(…) = 2

Callee context: 2

```
Number id(Number n) {
  return n;
}
```

Select(…) = 3

Callee context: 3

```
Number id(Number n) {
  return n;
}
```

Rule: Call

Caller context: c
Callee context: c^t

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\Rightarrow c' : o_i \in pt(c : x),$ $m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c' : o_i)$ $c'' : o_u \in pt(c : aj), 1 \le j \le n$ $c''' : o_v \in pt(c^t : m_{ret})$ ⟹ $c' : o_i \in pt(c^t : m_{this})$ $c'' : o_u \in pt(c^t : m_{pj}), 1 \le j \le n$ $c''' : o_v \in pt(c : r)$

- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- $\text{Select}(c, l, c' : o_i)$: selects context for target method m, based on the information available at call site l

➢ $c^t : m_{this}$: this variable of $c^t : m$

Rule: Call

Caller context: c
Callee context: c^t

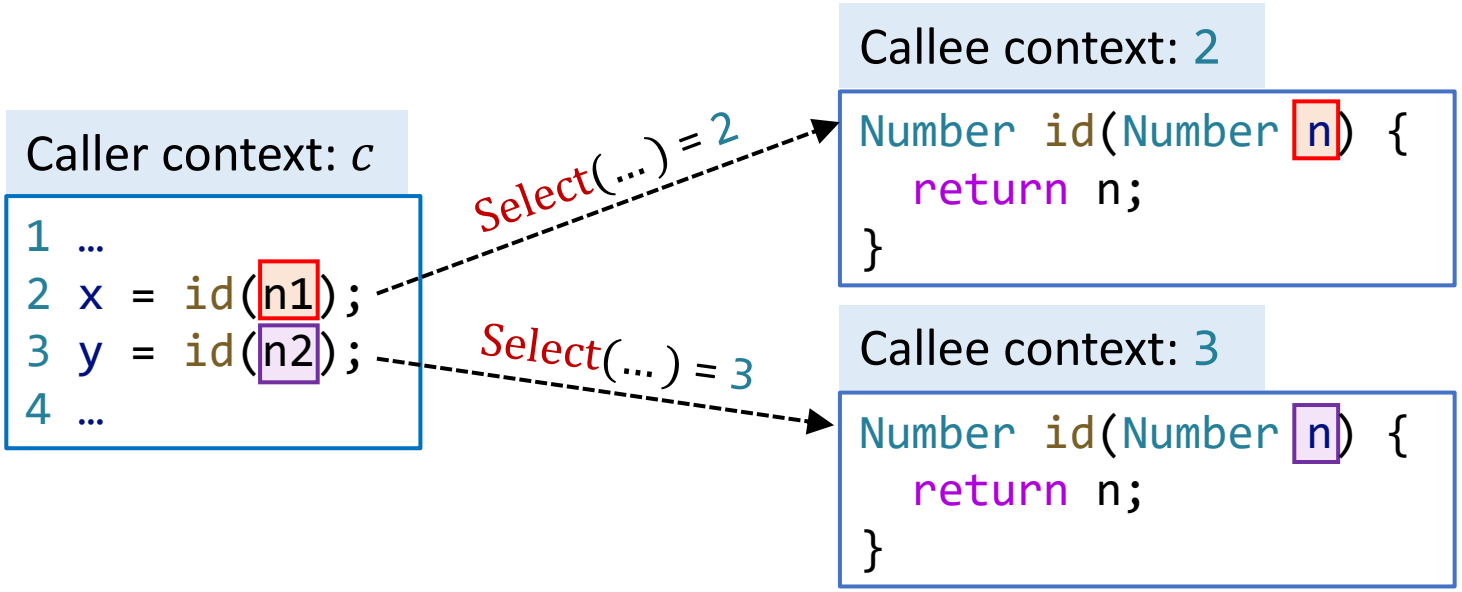
Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$c' : o_i \in pt(c : x)$, $m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c' : o_i)$ $c'' : o_u \in pt(c : aj), 1 \le j \le n$ $c''' : o_v \in pt(c^t : m_{ret})$ ⟹ $c' : o_i \in pt(c^t : m_{this})$ $c'' : o_u \in pt(c^t : m_{pj}), 1 \le j \le n$ $c''' : o_v \in pt(c : r)$

- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- $\text{Select}(c, l, c' : o_i)$: selects context for target method m, based on the information available at call site l
- $c^t : m_{this}$: this variable of $c^t : m$
- $c^t : m_{pj}$: the *j*-th parameter of $c^t : m$

Rule: Call

Caller context: c
Callee context: c^t

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$c' : o_i \in pt(c : x),$ $m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c' : o_i)$ $\Rightarrow c'' : o_u \in pt(c : aj), 1 \le j \le n$ $c''' : o_v \in pt(c^t : m_{ret})$ / $c' : o_i \in pt(c^t : m_{this})$ $\Rightarrow c'' : o_u \in pt(c^t : m_{pj}), 1 \le j \le n$ $c''' : o_v \in pt(c : r)$



Rule: Call

Caller context: c
Callee context: c^t

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c: x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(c: aj), 1 \le j \le n \\ \Rightarrow c''': o_v \in pt(c^t: m_{ret}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t: m_{this}) \\ c'': o_u \in pt(c^t: m_{pj}), 1 \le j \le n \\ \Rightarrow c''': o_v \in pt(c: r) \end{array}}$

- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- $\text{Select}(c, l, c': o_i)$: selects context for target method m, based on the information available at call site l
- $c^t: m_{this}$: this variable of $c^t: m$
- $c^t: m_{pj}$: the *j*-th parameter of $c^t: m$
- ➢ $c^t: m_{ret}$: the variable that holds the return value of $c^t: m$

Rule: Call

Caller context: c
Callee context: c^t

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c:x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(c:aj), 1 \le j \le n \\ \Longrightarrow c''': o_v \in pt(c^t: m_{ret}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t: m_{this}) \\ c'': o_u \in pt(c^t: m_{pj}), 1 \le j \le n \\ \Longrightarrow c''': o_v \in pt(c: r) \end{array}}$

Caller context: c

```
1 …
2 x = id(n1);
3 y = id(n2);
4 …
```

Select(…) = 2

Callee context: 2

```
Number id(Number n) {
  return n;
}
```

Select(…) = 3

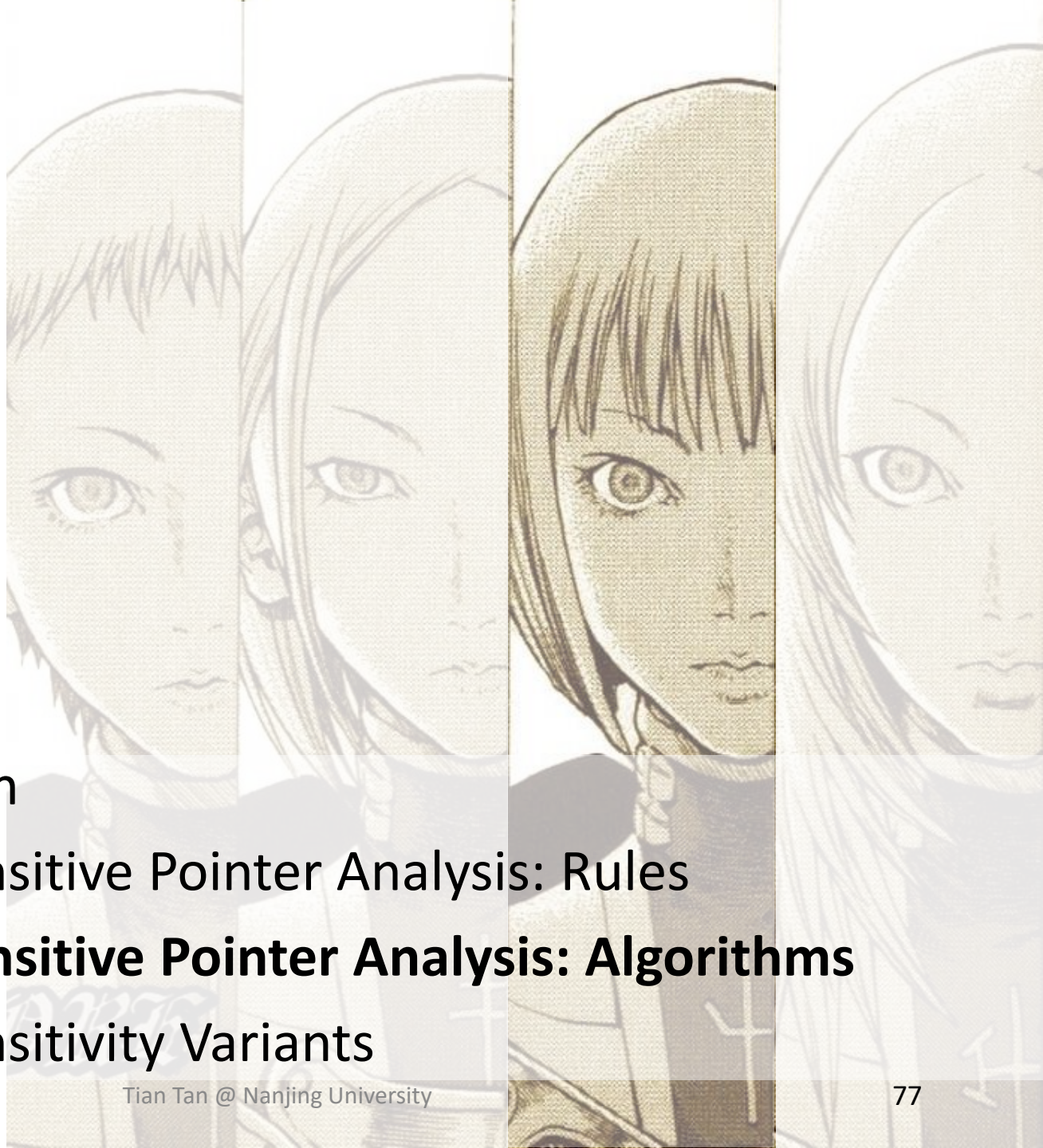
Callee context: 3

```
Number id(Number n) {
  return n;
}
```

The X You Need To Understand in This Lecture

- Concept of context sensitivity (C.S.)
- Concept of context-sensitive heap (C.S. heap)
- Why C.S. and C.S. heap improve precision
- Context-sensitive pointer analysis rules

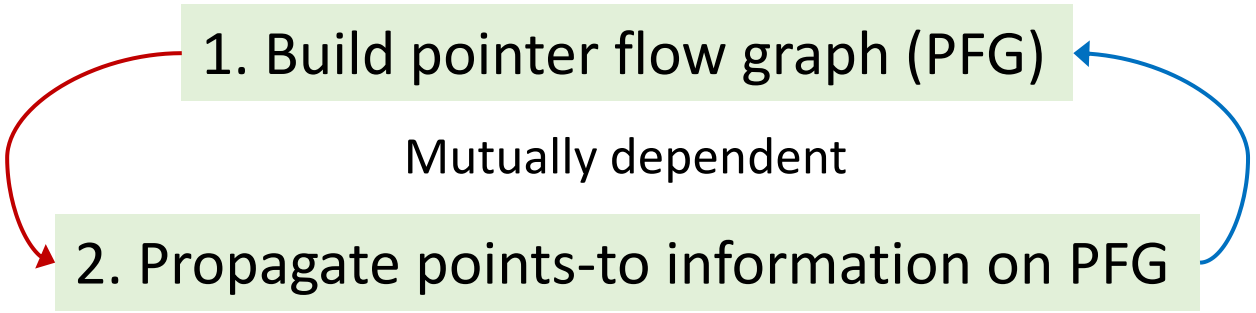
Contents



1. Introduction
2. Context Sensitive Pointer Analysis: Rules
3. **Context Sensitive Pointer Analysis: Algorithms**
4. Context Sensitivity Variants

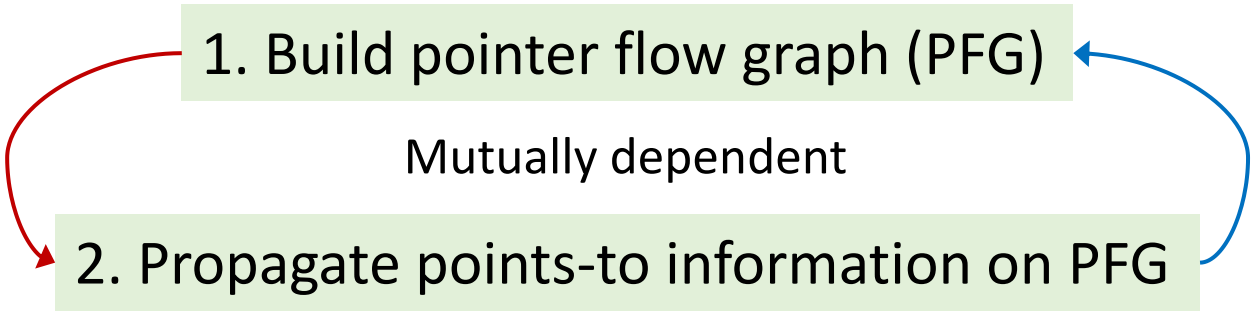
How to Implement Context-Sensitive Pointer Analysis

- Recall context-insensitive pointer analysis

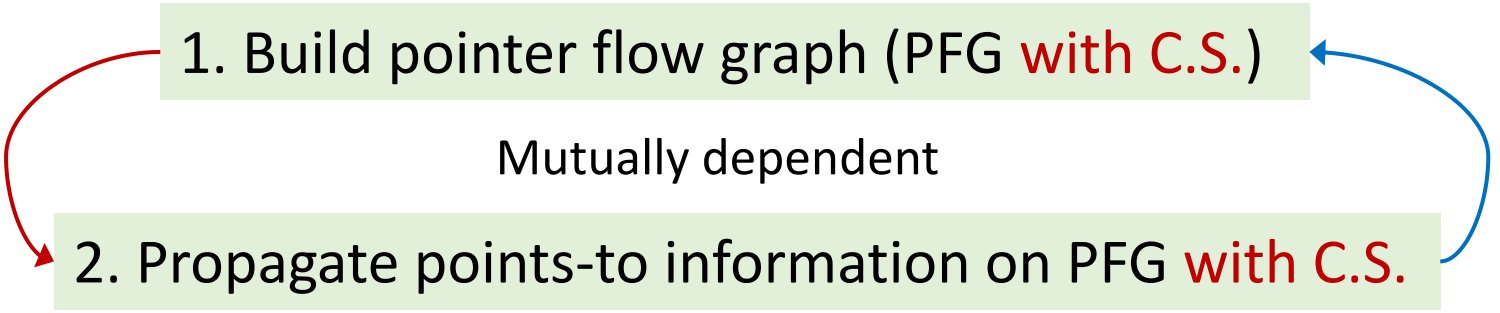


How to Implement Context-Sensitive Pointer Analysis

- Recall context-insensitive pointer analysis



- Context-sensitive pointer analysis



Pointer Flow Graph with C.S.

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program

Pointer Flow Graph with C.S.

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program

- Nodes: $\mathrm{CSPointer} = (\mathrm{C} \times \mathrm{V}) \cup (\mathrm{C} \times \mathrm{O} \times \mathrm{F})$

  A node *n* represents *a context-sensitive variable* or *a field of a context-sensitive abstract object*

  With C.S., the nodes (pointers) are qualified by contexts

Pointer Flow Graph with C.S.

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program

- Nodes: $\mathrm{CSPointer} = (\mathrm{C} \times \mathrm{V}) \cup (\mathrm{C} \times \mathrm{O} \times \mathrm{F})$

  A node n represents *a context-sensitive variable* or *a field of a context-sensitive abstract object*

  With C.S., the nodes (pointers) are qualified by contexts

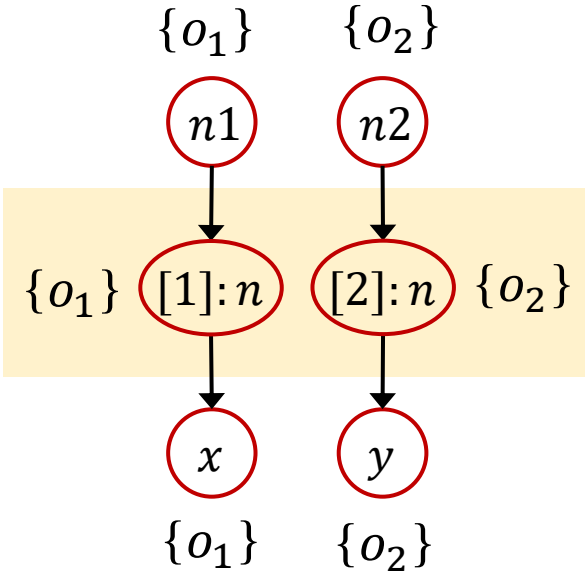
- Edges: $\mathrm{CSPointer} \times \mathrm{CSPointer}$

  An edge $x \rightarrow y$ means that the objects pointed by pointer x *may flow to* (and also be pointed to by) pointer y

e.g., edge $c{:}\,a \rightarrow c'{:}\,b$ means that objects in $pt(c{:}\,a)$ may flow to $pt(c'{:}\,b)$

An Example

```
1 x = id(n1);
2 y = id(n2);
3 int i = x.get();
4
5 Number id(Number n) {
6   return n;
7 }
```



The PFG contains two nodes for variable n in method `id()`, one node per context

Pointer Flow Graph: Edges

- PFG edges are added according to the statements of the program and the corresponding rules

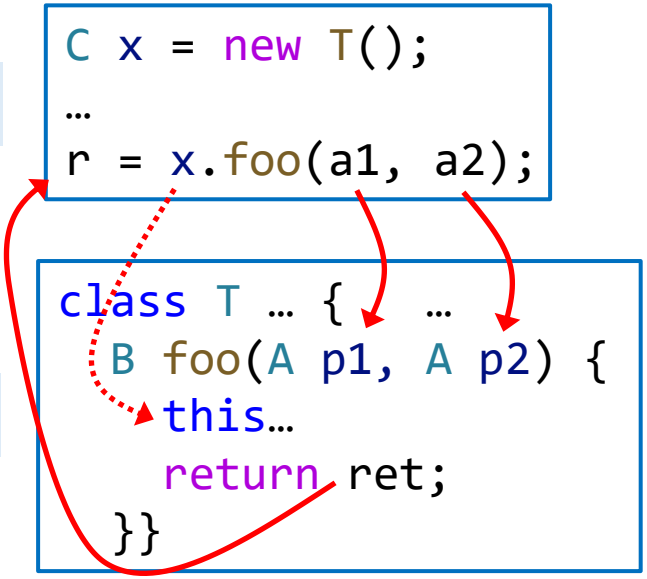
Kind	Statement	Rule (under context c)	PFG Edge
New	`i: x = new T()`	$\dfrac{}{c{:}\,o_i \in pt(c{:}\,x)}$	N/A
Assign	`x = y`	$\dfrac{c'{:}\,o_i \in pt(\boldsymbol{c{:}\,y})}{c'{:}\,o_i \in pt(\boldsymbol{c{:}\,x})}$	$c{:}\,x \leftarrow c{:}\,y$
Store	`x.f = y`	$\dfrac{c'{:}\,o_i \in pt(c{:}\,x), c''{:}\,o_j \in pt(\boldsymbol{c{:}\,y})}{c''{:}\,o_j \in pt(\boldsymbol{c'{:}\,o_i.f})}$	$c'{:}\,o_i.f \leftarrow c{:}\,y$
Load	`y = x.f`	$\dfrac{c'{:}\,o_i \in pt(c{:}\,x), c''{:}\,o_j \in pt(\boldsymbol{c'{:}\,o_i.f})}{c''{:}\,o_j \in pt(\boldsymbol{c{:}\,y})}$	$c{:}\,y \leftarrow c'{:}\,o_i.f$

Pointer Flow Graph: Call

Kind	Statement	Rule (under context c)
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} c': o_i \in pt(c:x), \\ m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i) \\ c'': o_u \in pt(\boldsymbol{c:aj}), 1 \le j \le n \\ c''': o_v \in pt(\boldsymbol{c^t: m_{ret}}) \end{array}}{\begin{array}{c} c': o_i \in pt(c^t: m_{this}) \\ c'': o_u \in pt(\boldsymbol{c^t: m_{pj}}), 1 \le j \le n \\ c''': o_v \in pt(\boldsymbol{c:r}) \end{array}}$

Caller context: c

Callee context: c^t



PFG Edge

$c: a1 \rightarrow c^t: m^{p1}$

…

$c: an \rightarrow c^t: m_{pn}$

$c: r \leftarrow c^t: m_{ret}$

C.S. Pointer Analysis: Algorithm

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y$, $c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f$, $c: y$)
        ProcessCall($c: x$, $c': o_i$)

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of **C.S.** reachable methods
CG	**C.S.** call graph edges

AddReachable($c: m$)
  if $c: m \notin RM$ **then**
    add $c: m$ to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle c: x, \{c: o_i\} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge($c: y$, $c: x$)

ProcessCall($c: x$, $c': o_i$)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\} \rangle$ to WL
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to CG
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}$, $c: r$)

C.I. Pointer Analysis: Algorithm

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable x **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of **C.I.** reachable methods
CG	**C.I.** call graph edges

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{ o_i \} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{ o_i \} \rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      AddReachable(: m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

C.S. Pointer Analysis: Algorithm

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y$, $c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f$, $c: y$)
        ProcessCall($c: x$, $c': o_i$)

S	Set of reachable statements
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C.S. Pointer Analysis: Algorithm

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S	Set of reachable statements
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Callee context: c^t

```
class T … {    …
  B foo(A p1, A p2) {
    this…
    return ret;
  }}
```

c^t: T.foo(A, A) $\in RM$

C.S. Pointer Analysis: Algorithm

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      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y, c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f, c: y$)
        ProcessCall($c: x, c': o_i$)

S	Set of reachable statements
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Caller context: c

```
1 C x = new T();
2 r = x.foo(a1, a2);
```

Callee context: c^t

```
class T … {
  B foo(A p1, A p2) {
    this…
    return ret;
  }}
```

c^t: T. foo(A, A) $\in RM$

$c: 2 \rightarrow c^t$: T. foo(A, A) $\in CG$

C.S. Pointer Analysis: Algorithm

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        foreach `x.f = y` $\in S$ **do**
          AddEdge($c{:}\,y$, $c'{:}\,o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c'{:}\,o_i.f$, $c{:}\,y$)
        ProcessCall($c{:}\,x$, $c'{:}\,o_i$)

AddReachable($c{:}\,m$)
  if $c{:}\,m \notin RM$ **then**
    add $c{:}\,m$ to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle c{:}\,x, \{c{:}\,o_i\} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge($c{:}\,y$, $c{:}\,x$)

Kind	Statement	Rule (under context c)	PFG Edge
New	`i: x = new T()`	$\dfrac{}{c{:}\,o_i \in pt(c{:}\,x)}$	N/A
Assign	`x = y`	$\dfrac{c'{:}\,o_i \in pt(\boldsymbol{c{:}\,y})}{c'{:}\,o_i \in pt(\boldsymbol{c{:}\,x})}$	$c{:}\,x \leftarrow c{:}\,y$

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    Propagate(n, Δ)
    if n represents a variable $c{:}\,x$ **then**
      foreach $c'{:}\,o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c{:}\,y$, $c'{:}\,o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c'{:}\,o_i.f$, $c{:}\,y$)
        ProcessCall($c{:}\,x$, $c'{:}\,o_i$)

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then**
    add $s \rightarrow t$ to PFG
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s) \rangle$ to WL

Propagate(n, pts)
  if pts is not empty **then**
    $pt(n) \cup= pts$
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts \rangle$ to WL

Exactly same as in C.I. analysis

S	Set of reachable statements
S_m	Set of statements in method m
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C.S. Pointer Analysis: Algorithm

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable $c{:}\,x$ **then**
      foreach $c'{:}\,o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c{:}\,y$, $c'{:}\,o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c'{:}\,o_i.f$, $c{:}\,y$)
        ProcessCall($c{:}\,x$, $c'{:}\,o_i$)

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then**
    add $s \rightarrow t$ to PFG
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s) \rangle$ to WL

Propagate(n, pts)
  if pts is not empty **then**
    $pt(n) \cup= pts$
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts \rangle$ to WL

Kind	Statement	Rule (under context c)	PFG Edge
Store	`x.f = y`	$\dfrac{c'{:}\,o_i \in pt(c{:}\,x),\ c''{:}\,o_j \in pt(\boldsymbol{c{:}\,y})}{c''{:}\,o_j \in pt(\boldsymbol{c'{:}\,o_i.f})}$	$c'{:}\,o_i.f \leftarrow c{:}\,y$
Load	`y = x.f`	$\dfrac{c'{:}\,o_i \in pt(c{:}\,x),\ c''{:}\,o_j \in pt(\boldsymbol{c'{:}\,o_i.f})}{c''{:}\,o_j \in pt(\boldsymbol{c{:}\,y})}$	$c{:}\,y \leftarrow c'{:}\,o_i.f$

C.S. Pointer Analysis: Algorithm

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y$, $c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f$, $c: y$)
        ProcessCall($c: x$, $c': o_i$)

ProcessCall($c: x$, $c': o_i$)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\} \rangle$ to WL
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to CG
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}$, $c: r$)

Kind	Statement	Rule (under context c)	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$c': o_i \in pt(c: x)$, $m = \text{Dispatch}(o_i, \text{k}), c^t = \text{Select}(c, l, c': o_i)$, $c'': o_u \in pt(\boldsymbol{c: aj}), 1 \le j \le n$, $c''': o_v \in pt(\boldsymbol{c^t: m_{ret}})$ / $c': o_i \in pt(c^t: m_{this})$, $c'': o_u \in pt(\boldsymbol{c^t: m_{pj}}), 1 \le j \le n$, $c''': o_v \in pt(\boldsymbol{c: r})$	$c: a1 \rightarrow c^t: m^{p1}$ … $c: an \rightarrow c^t: m_{pn}$, $c: r \leftarrow c^t: m_{ret}$

C.S. Pointer Analysis: Algorithm

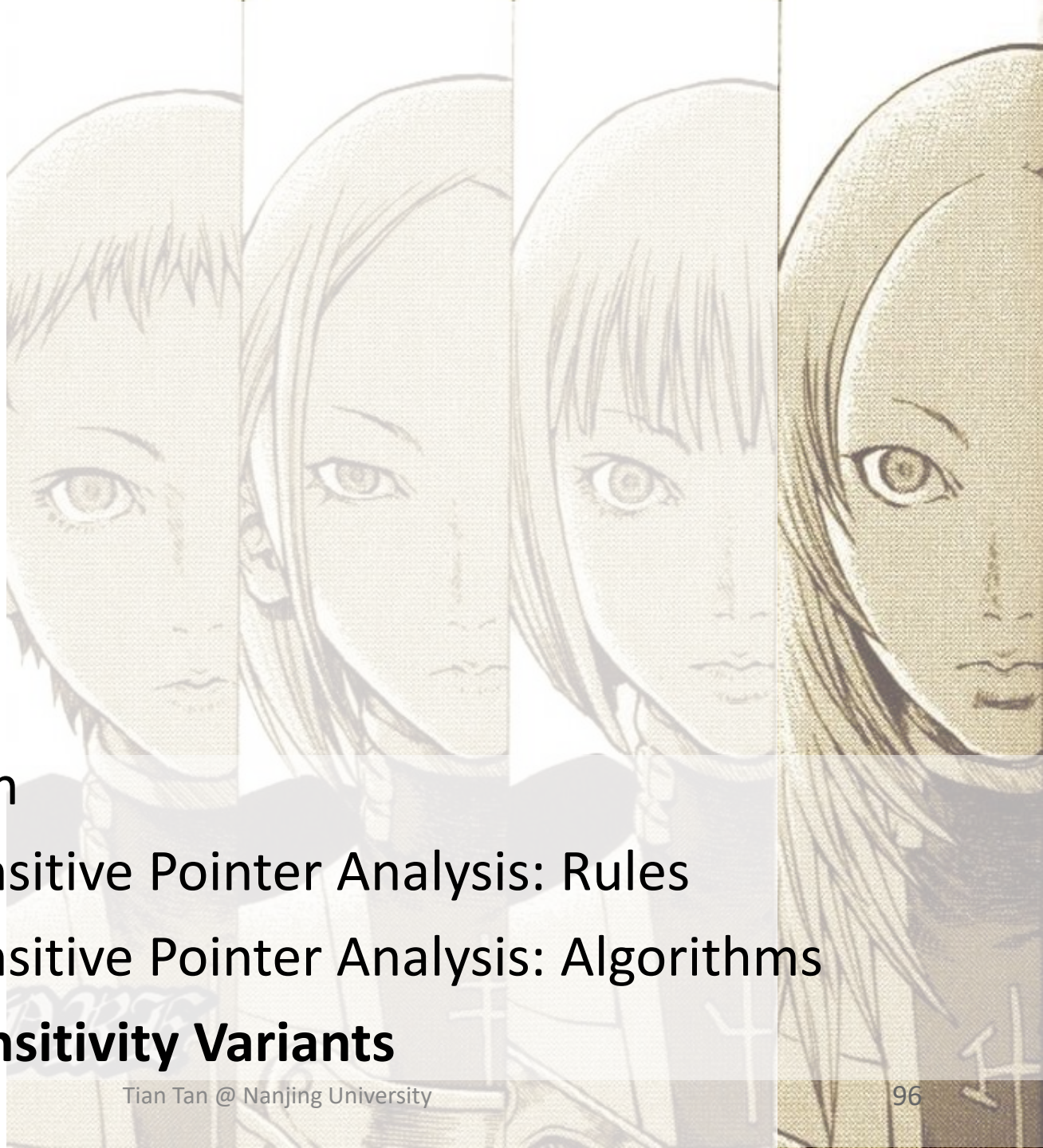
Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y$, $c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f$, $c: y$)
        ProcessCall($c: x$, $c': o_i$)

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of **C.S.** reachable methods
CG	**C.S.** call graph edges

AddReachable($c: m$)
  if $c: m \notin RM$ **then**
    add $c: m$ to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle c: x, \{c: o_i\} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge($c: y$, $c: x$)

ProcessCall($c: x$, $c': o_i$)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\} \rangle$ to WL
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to CG
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}$, $c: r$)

Contents



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2. Context Sensitive Pointer Analysis: Rules
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4. **Context Sensitivity Variants**

Context Sensitivity Variants

- Call-site sensitivity
- Object sensitivity
- Type sensitivity
- ……

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- Object sensitivity
- Type sensitivity
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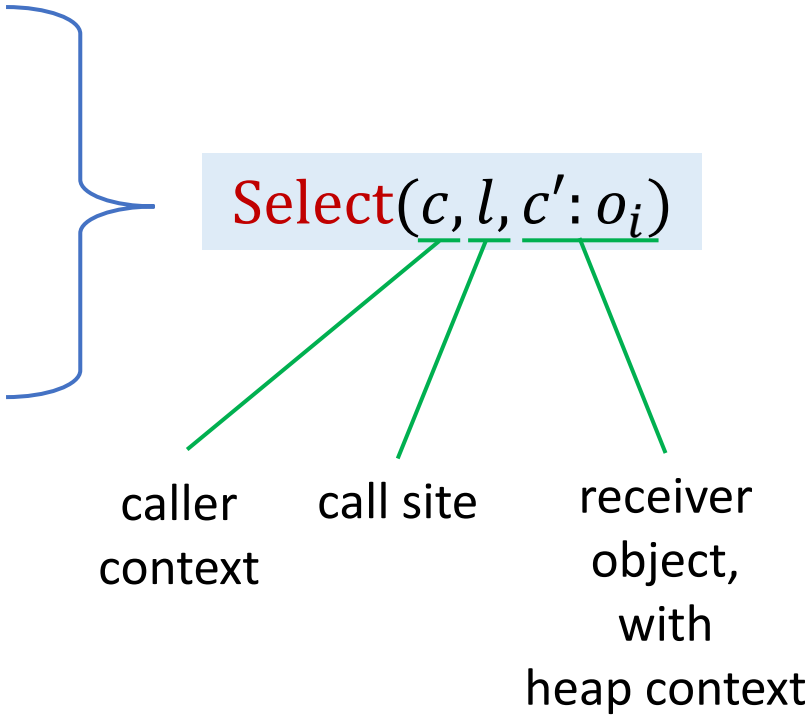
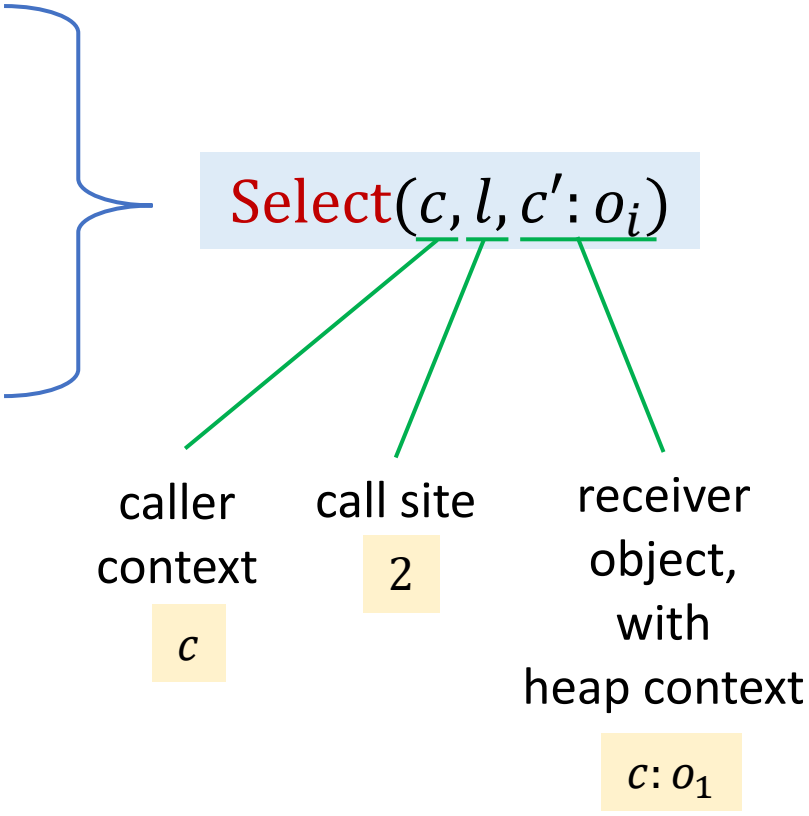


Context Sensitivity Variants

- Call-site sensitivity
- Object sensitivity
- Type sensitivity
- ……



Caller context: c

```
1 C x = new T();
2 r = x.foo(a1, a2);
```

Callee context: c^t

```
class T … {
  B foo(A p1, A p2) {
    this…
    return ret;
  }}
```

Context Insensitivity

- Can be seen as a special case of context sensitivity in C.S. analysis framework

$$\text{Select}(\_, \_, \_) = [\,]$$

Call-Site Sensitivity*

- Each context consists of a list of call sites (call chain)
  - At a method call, append the call site to the caller context as callee context
  - Essentially the abstraction of call stacks

$$\text{Select}(\boldsymbol{c}, \boldsymbol{l}, \_) = [l', \dots, l'', l]$$
where $c = [l', \dots, l'']$

Also called call-string sensitivity, or *k*-CFA

* Olin Shivers, 1991. "*Control-Flow Analysis of Higher-Order Languages*". Ph.D. Dissertation. Carnegie Mellon University.

Call-Site Sensitivity: Example

```
1  void main() {
2    …
3    a.foo();
4    …
5  }
6
7  void foo() {
8    …
9    b.bar();
10   …
11 }
12
13 void bar() {
14   …
15   …
16   …
17 }
```

Call-Site Sensitivity: Example

```
1  void main() {
2    …
3    a.foo();
4    …
5  }
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7  void foo() {
8    …
9    b.bar();
10   …
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13 void bar() {
14   …
15   …
16   …
17 }
```

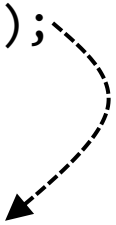
Context: [ ]

Call-Site Sensitivity: Example

```
1  void main() {
2    …
3    a.foo();
4    …
5  }
6
7  void foo() {
8    …
9    b.bar();
10   …
11 }
12
13 void bar() {
14   …
15   …
16   …
17 }
```

Context: [ ]

Context: [3]



Call-Site Sensitivity: Example

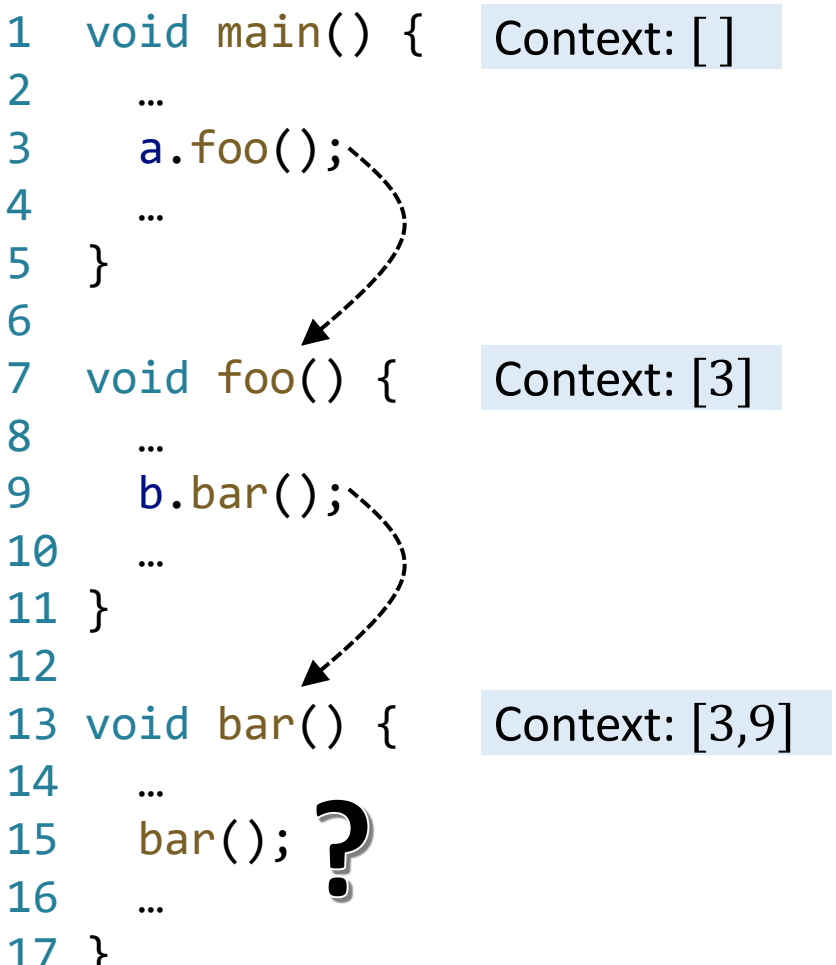
```
1  void main() {
2    …
3    a.foo();
4    …
5  }
6
7  void foo() {
8    …
9    b.bar();
10   …
11 }
12
13 void bar() {
14   …
15   …
16   …
17 }
```

Context: [ ]

Context: [3]

Context: [3,9]

Call-Site Sensitivity: Example



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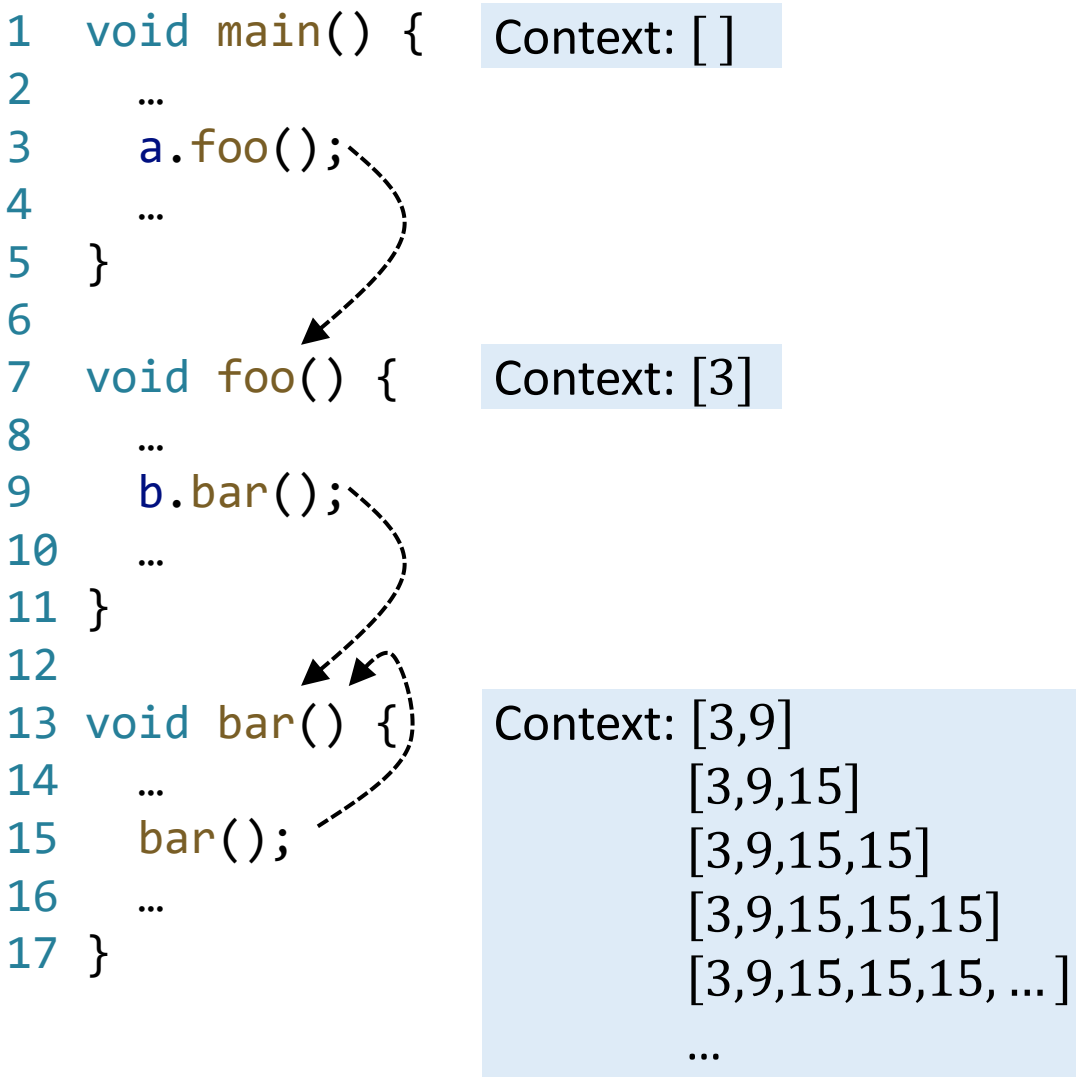

k-Limiting Context Abstraction

- Motivation
  - Ensure termination of pointer analysis
  - Avoid too many contexts (long call chains) in real-world programs blow up the pointer analysis

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- Motivation
  - Ensure termination of pointer analysis
  - Avoid too many contexts (long call chains) in real-world programs blow up the pointer analysis
- Approach: set an upper bound for length of contexts, denoted by *k*
  - For call-site sensitivity, each context consists of the last *k* call sites of the call chains
  - In practice, *k* is a small number (usually ≤3)
  - Method contexts and heap contexts may use different *k*
    - e.g., *k*=2 for method context, *k*=1 for heap contexts

k-Call-Site Sensitivity/*k*-CFA

- 1-call-site/1-CFA

$$\text{Select}(\_, \boldsymbol{l}, \_) = [l]$$

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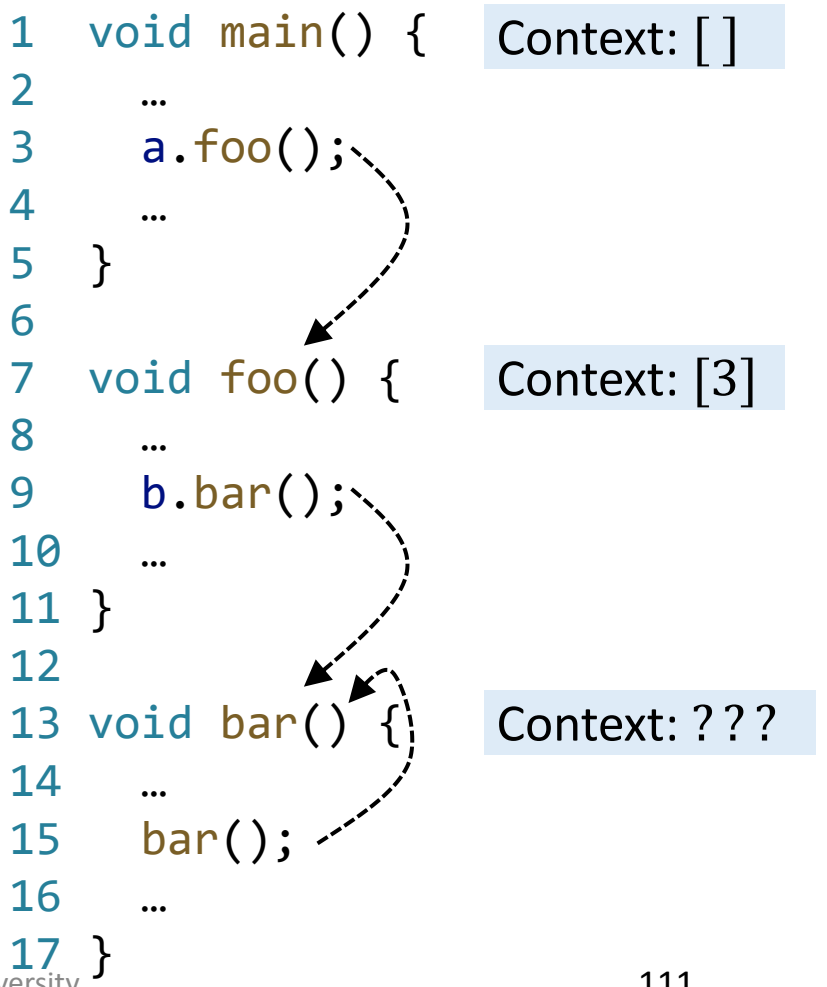

k-Call-Site Sensitivity/*k*-CFA

- 1-call-site/1-CFA

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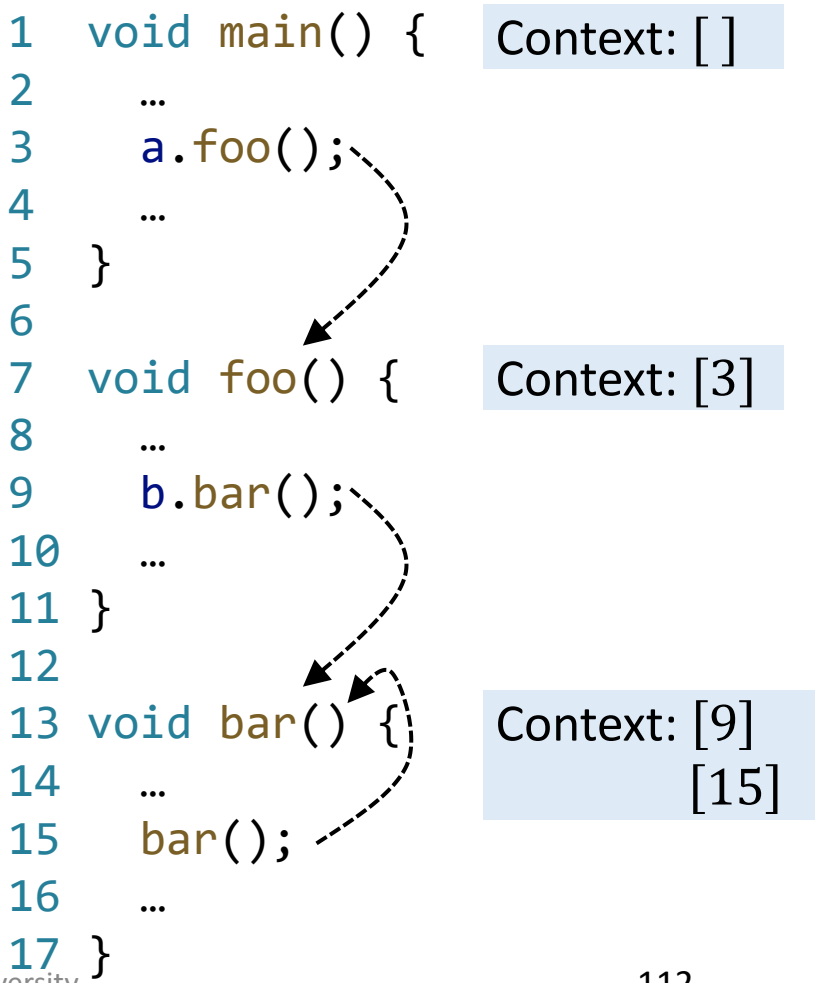

k-Call-Site Sensitivity/*k*-CFA

- 1-call-site/1-CFA

$$\text{Select}(\_, \boldsymbol{l}, \_) = [l]$$

- 2-call-site/2-CFA

$$\text{Select}(\boldsymbol{c}, \boldsymbol{l}, \_) = [l'', l]$$
where $c = [l', l'']$

1-Call-Site: Example

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  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
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1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
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5   }                          14     x = this.id(n1);
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5   }
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7   Number id(Number n) {
8     return n;
9   }
10  void m() {
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12    n1 = new One();
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```

```
interface Number {
  int get(); }
class One implements Number {
  public int get() { return 1; }}
class Two implements Number {
  public int get() { return 2; }}
```

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```

```
interface Number {
  int get(); }
class One implements Number {
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```

For simplicity, we do not apply C.S. heap and omit `this` variable of `C.id(Number)`

1-Call-Site: Example

WL: [ ]

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WL=[ ], *PFG*={}, *S*={}, *RM*={}, *CG*={}
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Processing:

PFG:

CG: { }

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1-Call-Site: Example

WL: [ ]

Solve(m^{entry})
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AddReachable([ ]: m^{entry})

Processing:

AddReachable(c: m)
if c: $m \notin RM$ **then**
add c: m to RM
$S \cup= S_m$
foreach `i: x = new T()` $\in S_m$ **do**
add $\langle c: x, \{c: o_i\}\rangle$ to WL
foreach `x = y` $\in S_m$ **do**
AddEdge(c: y, c: x)

PFG:

CG: { }

RM: { }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
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```

RM: { [ ]:C.main() }

1-Call-Site: Example

WL: [⟨[ ]:*c*, {o_3}⟩]

Solve(m^{entry})
WL=[ ], *PFG*={}, *S*={}, *RM*={}, *CG*={}
AddReachable([ ]: m^{entry})

Processing:

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if c: $m \notin RM$ **then**
add c: m to *RM*
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foreach `i: x = new T()` $\in S_m$ **do**
add ⟨c: x, {c: o_i}⟩ to *WL*
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Processing: $\langle$[ ]:c, $\{o_3\}\rangle$

PFG:

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Processing: $\langle$[ ]:c, $\{o_3\}\rangle$

$\{o_3\}$

PFG: [ ]:c

CG: { }

RM: { [ ]:C.main() }

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```

1-Call-Site: Example

WL: [ ]

ProcessCall($c: x, c': o_i$)
  foreach l: r = x.k(a1,…,an) ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i, c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

➡ ProcessCall($c: x, c': o_i$)

Processing: $\langle$[ ]:c, $\{o_3\}\rangle$

PFG:

$\{o_3\}$

[ ]:c

CG: { }

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    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

m = C.m()
c^t = ?

Processing: $\langle$[ ]:c, $\{o_3\}\rangle$

$\{o_3\}$

PFG: [ ]:c

ProcessCall($c: x$, c': o_i)

CG: { }

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      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
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m = C.m()
c^t = [4]

Processing: $\langle$[ ]:c, $\{o_3\}\rangle$

PFG:

$\{o_3\}$

[ ]:c

ProcessCall($c: x$, c$': o_i$)

CG: { }

```
1 class C {                  10   void m() {
2   static void main() {     11     Number n1,n2,x,y;
3     C c = new C();         12     n1 = new One();
4     c.m();                 13     n2 = new Two();
5   }                        14     x = this.id(n1);
6                            15     y = this.id(n2);
7   Number id(Number n) {    16     x.get();
8     return n;              17   }
9   }                        18 }
```

RM: { [ ]:C.main() }

1-Call-Site: Example

WL: [⟨[4]:C.m_{this}, {o_3}⟩]

ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add ⟨$c^t: m_{this}, \{c': o_i\}$⟩ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i, c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

m = C.m()
c^t = [4]

Processing: ⟨[ ]:c, {o_3}⟩

PFG: {o_3} [ ]:c

ProcessCall($c: x$, c$': o_i$)

CG: { }

```
1 class C {                  10   void m() {
2   static void main() {     11     Number n1,n2,x,y;
3     C c = new C();         12     n1 = new One();
4     c.m();                 13     n2 = new Two();
5   }                        14     x = this.id(n1);
6                            15     y = this.id(n2);
7   Number id(Number n) {    16     x.get();
8     return n;              17   }
9   }                        18 }
```

RM: { [ ]:C.main() }

1-Call-Site: Example

WL: $[\langle$[4]:C.m_{this}, $\{o_3\}\rangle]$

ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

m = C.m()
c^t = [4]

Processing: $\langle$[ ]:c, $\{o_3\}\rangle$

PFG:

$\{o_3\}$

[ ]:c

ProcessCall($c: x$, c$': o_i$)

CG: { [ ]:4 → [4]:C.m() }

```
1 class C {                  10   void m() {
2   static void main() {     11     Number n1,n2,x,y;
3     C c = new C();         12     n1 = new One();
4     c.m();                 13     n2 = new Two();
5   }                        14     x = this.id(n1);
6                            15     y = this.id(n2);
7   Number id(Number n) {    16     x.get();
8     return n;              17   }
9   }                        18 }
```

RM: { [ ]:C.main() }

1-Call-Site: Example

WL: [⟨[4]:C.m_{this}, {o_3}⟩, ⟨[4]:$n1$, {o_{12}}⟩, ⟨[4]:$n2$, {o_{13}}⟩]

ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add ⟨$c^t: m_{this}, \{c': o_i\}$⟩ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i, c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

ProcessCall($c: x, c': o_i$)

m = C.m()
c^t = [4]

Processing: ⟨[ ]:c, {o_3}⟩

PFG:

AddReachable($c: m$)
  if $c: m \notin RM$ **then**
    add $c: m$ to *RM*
    S ∪= S_m
    foreach `i: x = new T()` ∈ S_m **do**
      add ⟨$c: x$, {$c: o_i$}⟩ to *WL*
    foreach `x = y` ∈ S_m **do**
      AddEdge($c: y, c: x$)

```
1  class C {                    10    void m() {
2    static void main() {       11      Number n1,n2,x,y;
3      C c = new C();           12      n1 = new One();
4      c.m();                   13      n2 = new Two();
5    }                          14      x = this.id(n1);
6                               15      y = this.id(n2);
7    Number id(Number n) {      16      x.get();
8      return n;                17    }
9    }                          18  }
```

RM: { [ ]:C.main(), [4]:C.m() }

1-Call-Site: Example

WL: [⟨[4]:$n1, \{o_{12}\}$⟩, ⟨[4]:$n2, \{o_{13}\}$⟩]

Solve(m^{entry})
  WL=[ ], *PFG*={}, *S*={}, *RM*={}, *CG*={}
  AddReachable([ ]: m^{entry})
  while *WL* is not empty **do**
    remove ⟨n, pts⟩ from *WL*
    Δ = *pts* – *pt*(*n*)
    Propagate(*n*, Δ)
    if *n* represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y, c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f, c: y$)
        ProcessCall($c: x, c': o_i$)

Processing: ⟨[4]:C.$m_{this}, \{o_3\}$⟩

$\{o_3\}$

PFG: [ ]:c

$\{o_3\}$

[4]:C.m_{this}

CG: { [ ]:4 → [4]:C.m() }

RM: { [ ]:C.main(), [4]:C.m() }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14     x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16     x.get();
8     return n;                17   }
9   }                          18 }
```

1-Call-Site: Example

WL: [⟨[4]:$n1$, {o_{12}}⟩, ⟨[4]:$n2$, {o_{13}}⟩]

ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

➡ ProcessCall($c: x$, $c': o_i$)

Processing: ⟨[4]:C.m_{this}, {o_3}⟩

PFG:

{o_3}
[ ]:c

{o_3}
[4]:C.m_{this}

CG: { [ ]:4 → [4]:C.m() }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14 ➡   x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16     x.get();
8     return n;                17   }
9   }                          18 }
```

RM: { [ ]:C.main(), [4]:C.m() }

1-Call-Site: Example

WL: [⟨[4]:$n1, \{o_{12}\}$⟩, ⟨[4]:$n2, \{o_{13}\}$⟩]

ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

m = C.id(Number)
c^t = [14]

Processing: ⟨[4]:C.m_{this}, $\{o_3\}$⟩

PFG:

$\{o_3\}$
[ ]:c

$\{o_3\}$
[4]:C.m_{this}

ProcessCall($c: x$, c$': o_i$)

CG: { [ ]:4 → [4]:C.m() }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14     x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16     x.get();
8     return n;                17   }
9   }                          18 }
```

RM: { [ ]:C.main(), [4]:C.m() }

1-Call-Site: Example

WL: [⟨[4]:$n1$, {o_{12}}⟩, ⟨[4]:$n2$, {o_{13}}⟩]

ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
    add $c: l \rightarrow c^t: m$ to *CG*
    AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

m = C.id(Number)
c^t = [14]

Processing: ⟨[4]:C.m_{this}, {o_3}⟩

PFG:

{o_3}
[ ]:c

{o_3}
[4]:C.m_{this}

ProcessCall($c: x$, c$': o_i$)

CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number) }

```
1  class C {                    10   void m() {
2    static void main() {       11     Number n1,n2,x,y;
3      C c = new C();           12     n1 = new One();
4      c.m();                   13     n2 = new Two();
5    }                          14     x = this.id(n1);
6                               15     y = this.id(n2);
7    Number id(Number n) {      16     x.get();
8      return n;                17   }
9    }                          18 }
```

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number) }

1-Call-Site: Example

WL: [⟨[4]:$n1$, $\{o_{12}\}$⟩, ⟨[4]:$n2$, $\{o_{13}\}$⟩]

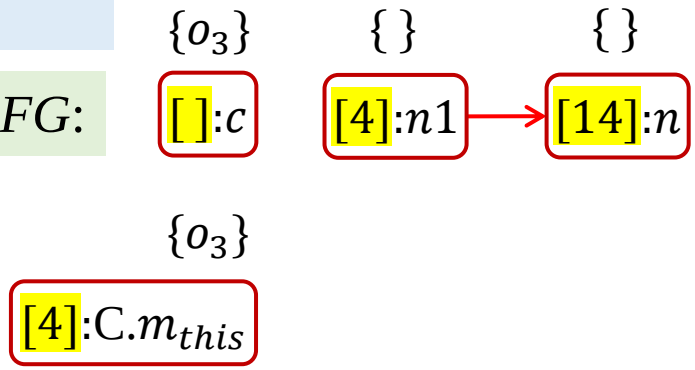
ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i, c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

ProcessCall($c: x, c': o_i$)

m = C.id(Number)
c^t = [14]

Processing: ⟨[4]:C.m_{this}, $\{o_3\}$⟩

PFG:



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number) }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14     x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16     x.get();
8     return n;                17   }
9   }                          18 }
```

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number) }

1-Call-Site: Example

WL: [⟨[4]:$n1$, $\{o_{12}\}$⟩, ⟨[4]:$n2$, $\{o_{13}\}$⟩]

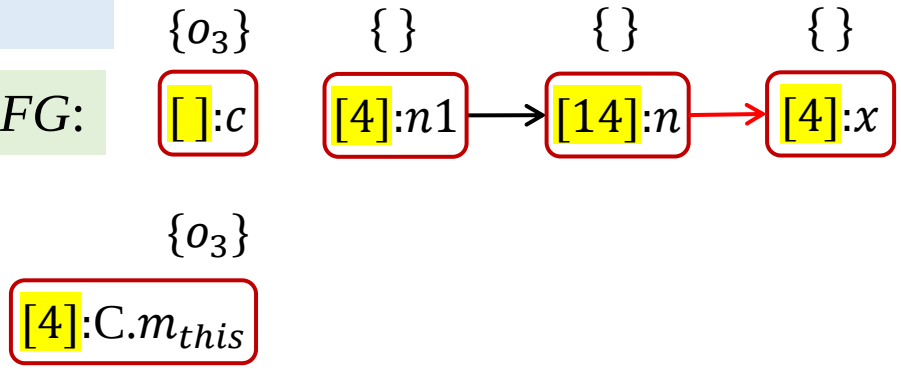
ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i, c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

ProcessCall($c: x, c': o_i$)

m = C.id(Number)
c^t = [14]

Processing: ⟨[4]:C.m_{this}, $\{o_3\}$⟩

PFG:



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number) }

```
1  class C {                    10    void m() {
2    static void main() {       11      Number n1,n2,x,y;
3      C c = new C();           12      n1 = new One();
4      c.m();                   13      n2 = new Two();
5    }                          14      x = this.id(n1);
6                               15      y = this.id(n2);
7    Number id(Number n) {      16      x.get();
8      return n;                17    }
9    }                          18  }
```

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number) }

1-Call-Site: Example

WL: [⟨[4]:$n1$, $\{o_{12}\}$⟩, ⟨[4]:$n2$, $\{o_{13}\}$⟩]

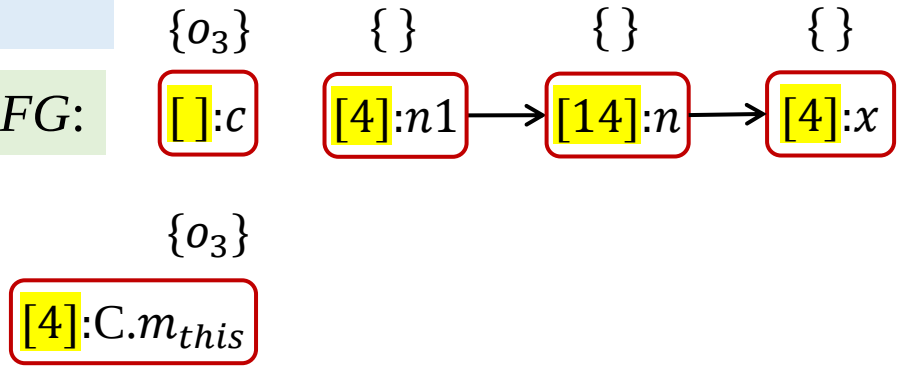
ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

ProcessCall($c: x$, $c': o_i$)

m = C.id(Number)
c^t = [15]

Processing: ⟨[4]:C.m_{this}, $\{o_3\}$⟩

PFG:



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number) }

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number),
[15]:C.id(Number) }

```
1  class C {                    10    void m() {
2    static void main() {       11      Number n1,n2,x,y;
3      C c = new C();           12      n1 = new One();
4      c.m();                   13      n2 = new Two();
5    }                          14      x = this.id(n1);
6                               15      y = this.id(n2);
7    Number id(Number n) {      16      x.get();
8      return n;                17    }
9    }                          18  }
```

1-Call-Site: Example

WL: [⟨[4]:$n1$, {o_{12}}⟩, ⟨[4]:$n2$, {o_{13}}⟩]

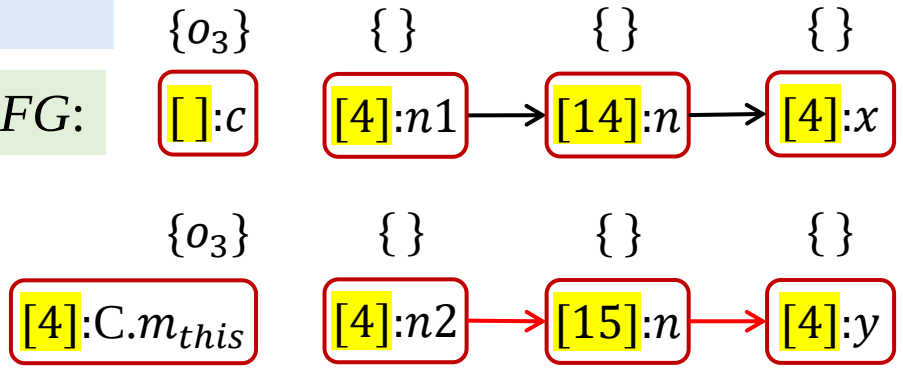
ProcessCall($c: x, c': o_i$)
  foreach `l: r = x.k(a1,…,an)` ∈ S **do**
    m = Dispatch(o_i, k)
    c^t = Select($c, l, c': o_i$)
    add $\langle c^t: m_{this}, \{c': o_i\}\rangle$ to *WL*
    if $c: l \rightarrow c^t: m \notin CG$ **then**
      add $c: l \rightarrow c^t: m$ to *CG*
      AddReachable($c^t: m$)
      foreach parameter p_i of m **do**
        AddEdge($c: a_i$, $c^t: p_i$)
      AddEdge($c^t: m_{ret}, c: r$)

ProcessCall($c: x$, $c': o_i$)

m = C.id(Number)
c^t = [15]

Processing: ⟨[4]:C.m_{this}, {o_3}⟩

PFG:



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number) }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14     x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16     x.get();
8     return n;                17   }
9   }                          18 }
```

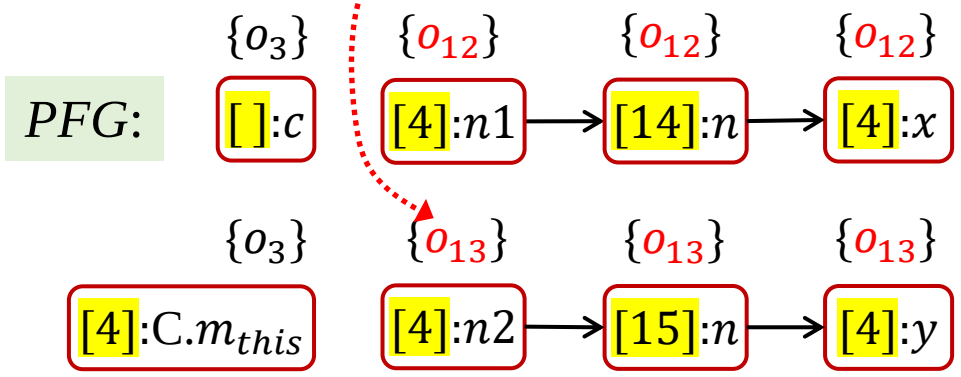
RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number),
[15]:C.id(Number) }

1-Call-Site: Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    Δ = pts – $pt(n)$
    Propagate(n, Δ)
    if n represents a variable c: x **then**
      foreach c': $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(c: y, c': $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge(c': $o_i.f$, c: y)
        ProcessCall(c: x, c': o_i)

WL: [⟨[4]:$n1$, {o_{12}}⟩, ⟨[4]:$n2$, {o_{13}}⟩]

Processing: …



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number) }

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number),
[15]:C.id(Number) }

```
1  class C {                    10    void m() {
2    static void main() {       11      Number n1,n2,x,y;
3      C c = new C();           12      n1 = new One();
4      c.m();                   13      n2 = new Two();
5    }                          14      x = this.id(n1);
6                               15      y = this.id(n2);
7    Number id(Number n) {      16      x.get();
8      return n;                17    }
9    }                          18  }
```

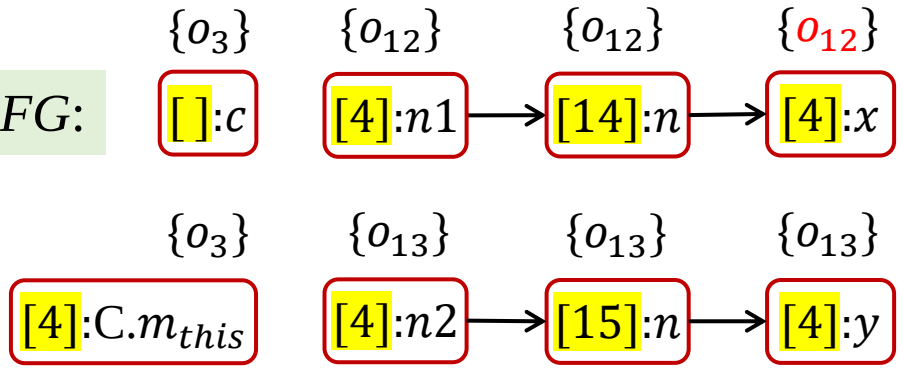
1-Call-Site: Example

WL: [ ]

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while *WL* is not empty **do**
    remove $\langle n, pts \rangle$ from *WL*
    Δ = *pts* – *pt*(*n*)
    Propagate(*n*, Δ)
    if *n* represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y, c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f, c: y$)
      ⟹ ProcessCall($c: x, c': o_i$)

Processing: $\langle$[4]: $x, \{o_{12}\}\rangle$

PFG:



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number),
[4]:16 → [16]:One.get() }

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number),
[15]:C.id(Number),
[16]:One.get() }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14     x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16 ⟹  x.get();
8     return n;                17   }
9   }                          18 }
```

1-Call-Site: Example

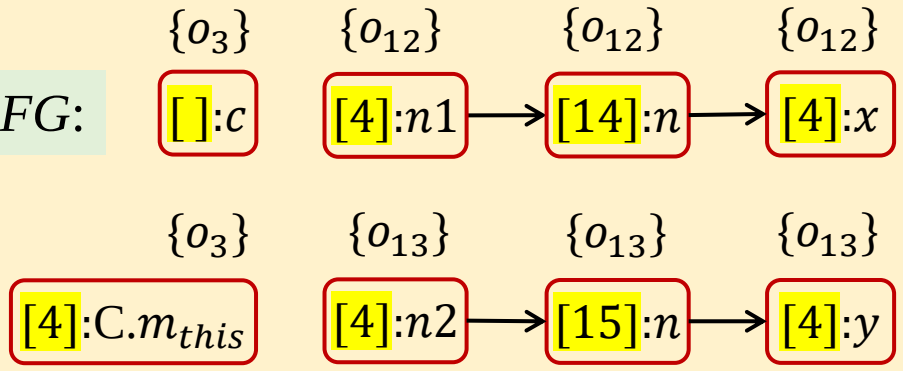
WL: [ ]

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable([ ]: m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable $c: x$ **then**
      foreach $c': o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge($c: y$, $c': o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($c': o_i.f$, $c: y$)
        ProcessCall($c: x$, $c': o_i$)

Algorithm finishes

Final results

PFG:



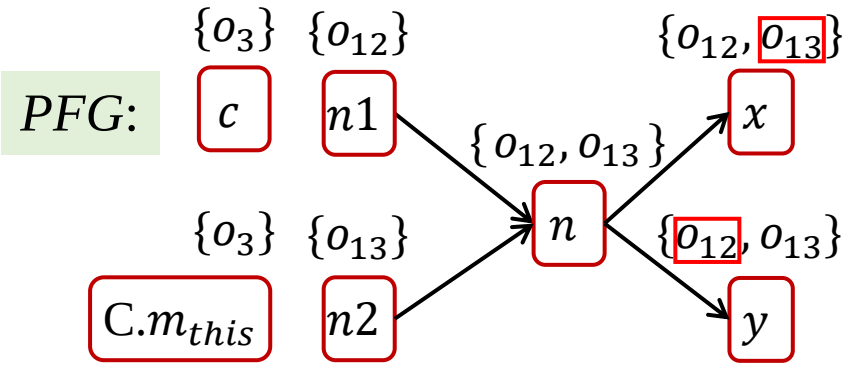
CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number),
[4]:16 → [16]:One.get() }

RM: { [ ]:C.main(), [4]:C.m(),
[14]:C.id(Number),
[15]:C.id(Number),
[16]:One.get() }

```
1 class C {
2   static void main() {
3     C c = new C();
4     c.m();
5   }
6
7   Number id(Number n) {
8     return n;
9   }
```

```
10   void m() {
11     Number n1,n2,x,y;
12     n1 = new One();
13     n2 = new Two();
14     x = this.id(n1);
15     y = this.id(n2);
16     x.get();
17   }
18 }
```

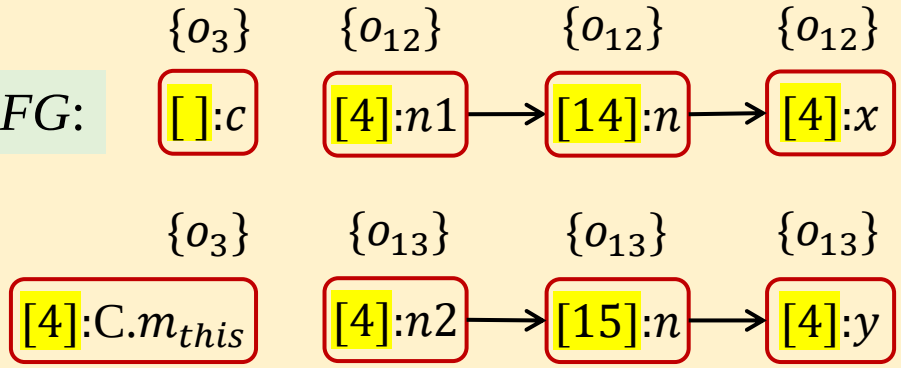
C.I. vs. C.S. (1-Call-Site)



CG: { 4 → C.m(),
14 → C.id(Number),
15 → C.id(Number),
16 → One.get(),
16 → Two.get() }

Spurious

Final results



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number),
[4]:16 → [16]:One.get() }

```
1 class C {                    10   void m() {
2   static void main() {       11     Number n1,n2,x,y;
3     C c = new C();           12     n1 = new One();
4     c.m();                   13     n2 = new Two();
5   }                          14     x = this.id(n1);
6                              15     y = this.id(n2);
7   Number id(Number n) {      16     x.get();
8     return n;                17   }
9   }                          18 }
```

Object Sensitivity*

- Each context consists of a list of abstract objects (represented by their allocation sites)
  - At a method call, use the receiver object with its heap context as callee context
  - Distinguish the operations of data flow on different objects

$$\text{Select}(\_, \_, \boldsymbol{c'}\!:\boldsymbol{o_i}) = [o_j, \dots, o_k, o_i]$$
$$\text{where } c' = [o_j, \dots, o_k]$$

Essentially "allocation-site sensitivity"

* Ana Milanova, Atanas Rountev, and Barbara G. Ryder. "*Parameterized Object Sensitivity for Points-to and Side-Effect Analyses for Java*". ISSTA 2002.

Object Sensitivity: Example

```
1  a1 = new A();
2  a2 = new A();
3  b1 = new B();
4  b2 = new B();
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

Object Sensitivity: Example

```
1  a1 = new A(); a1 → o1
2  a2 = new A(); a2 → o2
3  b1 = new B(); b1 → o3
4  b2 = new B(); b2 → o4
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

Object Sensitivity: Example

```
1  a1 = new A(); a1 → o1
2  a2 = new A(); a2 → o2
3  b1 = new B(); b1 → o3
4  b2 = new B(); b2 → o4
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

$c^t = ?$

Object Sensitivity: Example

```
1  a1 = new A();  a1 → o1
2  a2 = new A();  a2 → o2
3  b1 = new B();  b1 → o3
4  b2 = new B();  b2 → o4
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

$c^t = [o_1]$

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3

Object Sensitivity: Example

```
1  a1 = new A();  a1 → o1
2  a2 = new A();  a2 → o2
3  b1 = new B();  b1 → o3
4  b2 = new B();  b2 → o4
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

$c^t = [o_1]$

$c^t = ?$

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3

Object Sensitivity: Example

```
1   a1 = new A();   a1 → o1
2   a2 = new A();   a2 → o2
3   b1 = new B();   b1 → o3
4   b2 = new B();   b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [o_1]$

$c^t = [o_1]$

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3
$[o_1]: doSet_{this}$	o_1
$[o_1]: p$	o_3
Field	**Object**
$o_1.\mathtt{f}$	o_3

Object Sensitivity: Example

```
1   a1 = new A();  a1 → o1
2   a2 = new A();  a2 → o2
3   b1 = new B();  b1 → o3
4   b2 = new B();  b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [o_1]$

$c^t = [o_2]$

$c^t = [o_1]$

$c^t = [o_2]$

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3
$[o_1]: doSet_{this}$	o_1
$[o_1]: p$	o_3
$[o_2]: set_{this}$	o_2
$[o_2]: b$	o_4
$[o_2]: doSet_{this}$	o_2
$[o_2]: p$	o_4
Field	**Object**
$o_1.\mathtt{f}$	o_3
$o_2.\mathtt{f}$	o_4

Object Sensitivity: Example

```
1   a1 = new A();  a1 → o1
2   a2 = new A();  a2 → o2
3   b1 = new B();  b1 → o3
4   b2 = new B();  b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

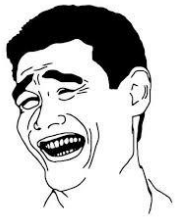
$c^t = [o_1]$

$c^t = [o_2]$

$c^t = [o_1]$

$c^t = [o_2]$

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3
$[o_1]: doSet_{this}$	o_1
$[o_1]: p$	o_3
$[o_2]: set_{this}$	o_2
$[o_2]: b$	o_4
$[o_2]: doSet_{this}$	o_2
$[o_2]: p$	o_4
x	o_3
Field	**Object**
$o_1.\mathtt{f}$	o_3
$o_2.\mathtt{f}$	o_4



Call-Site vs. Object Sensitivity

1-call-site

1-object

```
1   a1 = new A();  a1 → o1
2   a2 = new A();  a2 → o2
3   b1 = new B();  b1 → o3
4   b2 = new B();  b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3
$[o_1]: doSet_{this}$	o_1
$[o_1]: p$	o_3
$[o_2]: set_{this}$	o_2
$[o_2]: b$	o_4
$[o_2]: doSet_{this}$	o_2
$[o_2]: p$	o_4
x	o_3
Field	**Object**
$o_1.\mathtt{f}$	o_3
$o_2.\mathtt{f}$	o_4

Call-Site vs. Object Sensitivity

```
1  a1 = new A();  a1 → o1
2  a2 = new A();  a2 → o2
3  b1 = new B();  b1 → o3
4  b2 = new B();  b2 → o4
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

$c^t = [5]$

1-call-site

Variable	Object
$[5]: set_{this}$	o_1
$[5]: b$	o_3

1-object

Variable	Object
$[o_1]: set_{this}$	o_1
$[o_1]: b$	o_3
$[o_1]: doSet_{this}$	o_1
$[o_1]: p$	o_3
$[o_2]: set_{this}$	o_2
$[o_2]: b$	o_4
$[o_2]: doSet_{this}$	o_2
$[o_2]: p$	o_4
x	o_3
Field	**Object**
$o_1.f$	o_3
$o_2.f$	o_4

Call-Site vs. Object Sensitivity

```
1  a1 = new A();  a1 → o1
2  a2 = new A();  a2 → o2
3  b1 = new B();  b1 → o3
4  b2 = new B();  b2 → o4
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```

$c^t = [5]$

$c^t = [12]$

1-call-site

Variable	Object
[5]: set_{this}	o_1
[5]: b	o_3
[12]: $doSet_{this}$	o_1
[12]: p	o_3
Field	**Object**
o_1.f	o_3

1-object

Variable	Object
$[o_1]$: set_{this}	o_1
$[o_1]$: b	o_3
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	o_3
$[o_2]$: set_{this}	o_2
$[o_2]$: b	o_4
$[o_2]$: $doSet_{this}$	o_2
$[o_2]$: p	o_4
x	o_3
Field	**Object**
o_1.f	o_3
o_2.f	o_4

Call-Site vs. Object Sensitivity

```
1   a1 = new A();   a1 → o1
2   a2 = new A();   a2 → o2
3   b1 = new B();   b1 → o3
4   b2 = new B();   b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [5]$

$c^t = [6]$

$c^t = [12]$

1-call-site

Variable	Object
[5]: set_{this}	o_1
[5]: b	o_3
[6]: set_{this}	o_2
[6]: b	o_4
[12]: $doSet_{this}$	o_1
[12]: p	o_3
Field	**Object**
$o_1.f$	o_3

1-object

Variable	Object
$[o_1]$: set_{this}	o_1
$[o_1]$: b	o_3
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	o_3
$[o_2]$: set_{this}	o_2
$[o_2]$: b	o_4
$[o_2]$: $doSet_{this}$	o_2
$[o_2]$: p	o_4
x	o_3
Field	**Object**
$o_1.f$	o_3
$o_2.f$	o_4

Call-Site vs. Object Sensitivity

```
1   a1 = new A();   a1 → o1
2   a2 = new A();   a2 → o2
3   b1 = new B();   b1 → o3
4   b2 = new B();   b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [5]$

$c^t = [6]$

$c^t = [12]$

$c^t = ?$

1-call-site

Variable	Object
[5]: set_{this}	o_1
[5]: b	o_3
[6]: set_{this}	o_2
[6]: b	o_4
[12]: $doSet_{this}$	o_1
[12]: p	o_3
Field	**Object**
$o_1.f$	o_3

1-object

Variable	Object
$[o_1]$: set_{this}	o_1
$[o_1]$: b	o_3
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	o_3
$[o_2]$: set_{this}	o_2
$[o_2]$: b	o_4
$[o_2]$: $doSet_{this}$	o_2
$[o_2]$: p	o_4
x	o_3
Field	**Object**
$o_1.f$	o_3
$o_2.f$	o_4

Call-Site vs. Object Sensitivity

```
1   a1 = new A();   a1 → o1
2   a2 = new A();   a2 → o2
3   b1 = new B();   b1 → o3
4   b2 = new B();   b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [5]$

$c^t = [6]$

$c^t = [12]$

1-call-site

Variable	Object
[5]: set_{this}	o_1
[5]: b	o_3
[6]: set_{this}	o_2
[6]: b	o_4
[12]: $doSet_{this}$	o_1, o_2
[12]: p	o_3, o_4
Field	**Object**
$o_1.f$	o_3, o_4
$o_2.f$	o_3, o_4

1-object

Variable	Object
$[o_1]$: set_{this}	o_1
$[o_1]$: b	o_3
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	o_3
$[o_2]$: set_{this}	o_2
$[o_2]$: b	o_4
$[o_2]$: $doSet_{this}$	o_2
$[o_2]$: p	o_4
x	o_3
Field	**Object**
$o_1.f$	o_3
$o_2.f$	o_4

Call-Site vs. Object Sensitivity

```
1   a1 = new A();  a1 → o1
2   a2 = new A();  a2 → o2
3   b1 = new B();  b1 → o3
4   b2 = new B();  b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [5]$

$c^t = [6]$

$c^t = [12]$

1-call-site

Variable	Object
[5]: set_{this}	o_1
[5]: b	o_3
[6]: set_{this}	o_2
[6]: b	o_4
[12]: $doSet_{this}$	o_1, o_2
[12]: p	o_3, o_4
x	o_3, o_4
Field	**Object**
o_1.f	o_3, o_4
o_2.f	o_3, o_4

1-object

Variable	Object
[o_1]: set_{this}	o_1
[o_1]: b	o_3
[o_1]: $doSet_{this}$	o_1
[o_1]: p	o_3
[o_2]: set_{this}	o_2
[o_2]: b	o_4
[o_2]: $doSet_{this}$	o_2
[o_2]: p	o_4
x	o_3
Field	**Object**
o_1.f	o_3
o_2.f	o_4

Call-Site vs. Object Sensitivity

```
1   a1 = new A();   a1 → o1
2   a2 = new A();   a2 → o2
3   b1 = new B();   b1 → o3
4   b2 = new B();   b2 → o4
5   a1.set(b1);
6   a2.set(b2);
7   x = a1.get();
8
9   class A {
10    B f;
11    void set(B b) {
12      this.doSet(b);
13    }
14    void doSet(B p) {
15      this.f = p;
16    }
17    B get() {
18      return this.f;
19    }
20  }
```

$c^t = [5]$

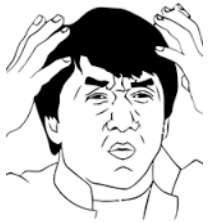
$c^t = [6]$

$c^t = [12]$

1-call-site

Variable	Object
[5]: set_{this}	o_1
[5]: b	o_3
[6]: set_{this}	o_2
[6]: b	o_4
[12]: $doSet_{this}$	o_1, o_2
[12]: p	o_3, o_4
x	o_3, o_4
Field	**Object**
$o_1.f$	o_3, o_4
$o_2.f$	o_3, o_4

Spurious

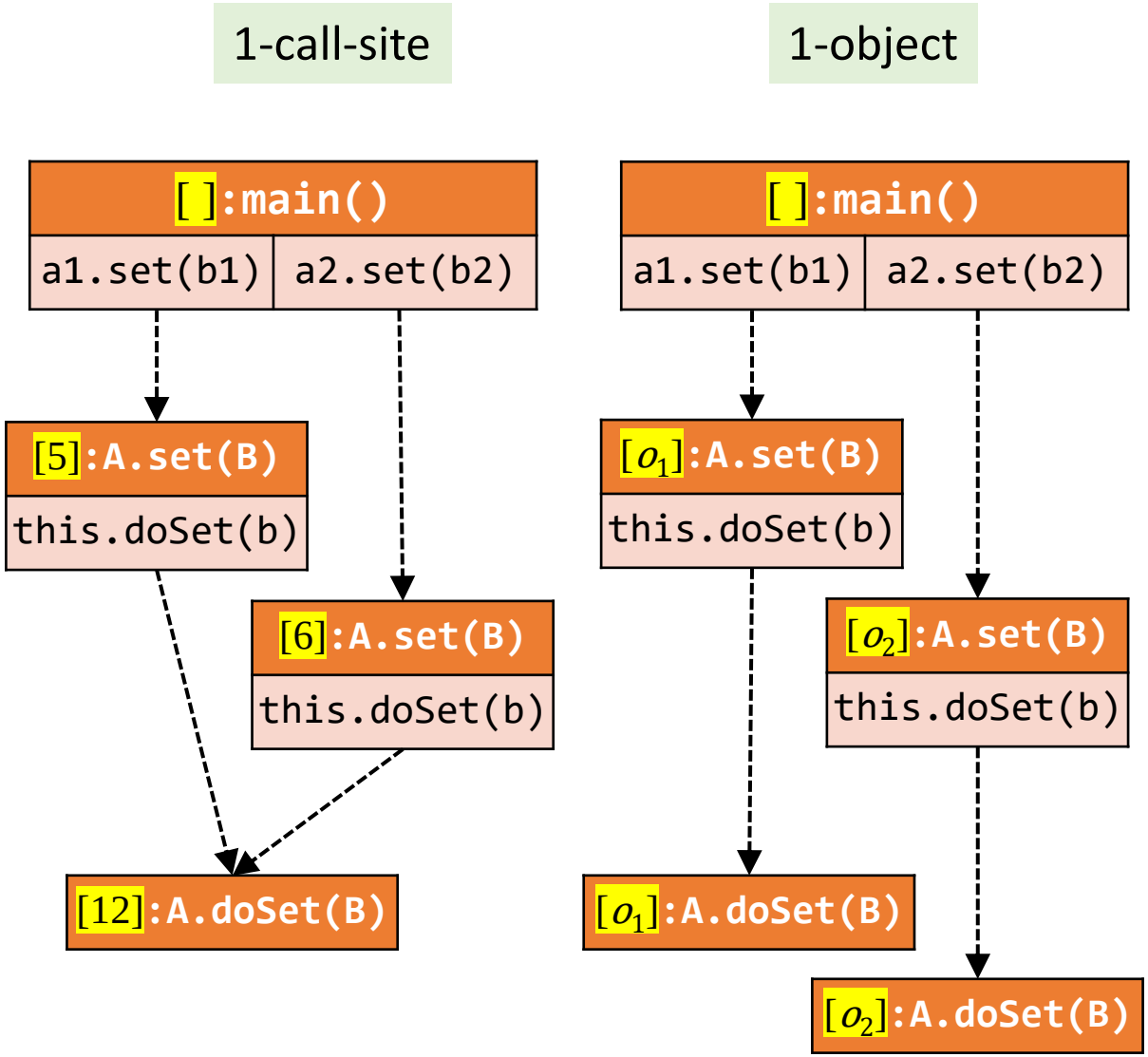


1-object

Variable	Object
$[o_1]$: set_{this}	o_1
$[o_1]$: b	o_3
$[o_1]$: $doSet_{this}$	o_1
$[o_1]$: p	o_3
$[o_2]$: set_{this}	o_2
$[o_2]$: b	o_4
$[o_2]$: $doSet_{this}$	o_2
$[o_2]$: p	o_4
x	o_3
Field	**Object**
$o_1.f$	o_3
$o_2.f$	o_4

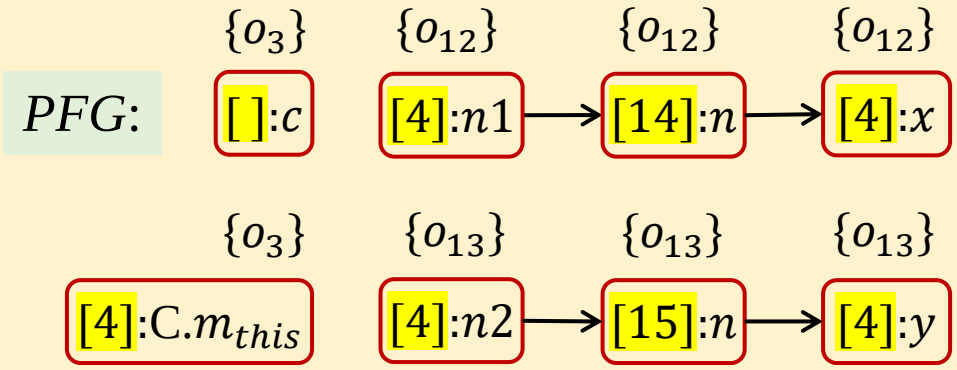
Call-Site vs. Object Sensitivity

```
1  a1 = new A();
2  a2 = new A();
3  b1 = new B();
4  b2 = new B();
5  a1.set(b1);
6  a2.set(b2);
7  x = a1.get();
8
9  class A {
10   B f;
11   void set(B b) {
12     this.doSet(b);
13   }
14   void doSet(B p) {
15     this.f = p;
16   }
17   B get() {
18     return this.f;
19   }
20 }
```



C.S. (1-Object) vs. C.S. (1-Call-Site)

Final results



CG: { [ ]:4 → [4]:C.m(),
[4]:14 → [14]:C.id(Number),
[4]:15 → [15]:C.id(Number),
[4]:16 → [16]:One.get() }

```
1 class C {                  10   void m() {
2   static void main() {     11     Number n1,n2,x,y;
3     C c = new C();         12     n1 = new One();
4     c.m();                 13     n2 = new Two();
5   }                        14     x = this.id(n1);
6                            15     y = this.id(n2);
7   Number id(Number n) {    16     x.get();
8     return n;              17   }
9   }                        18 }
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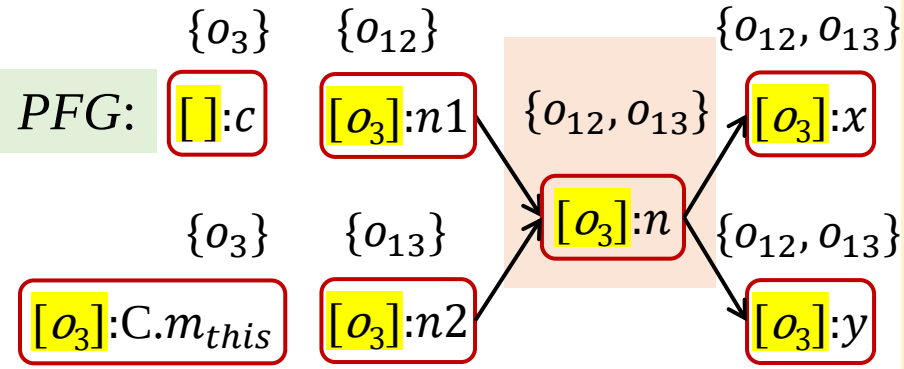
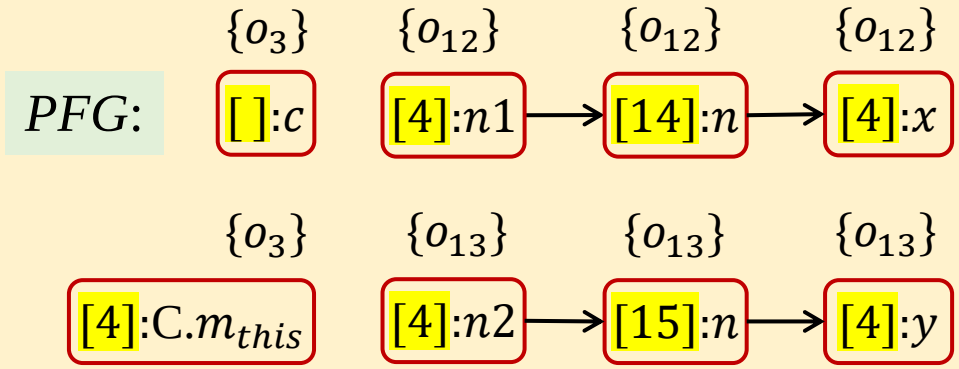
C.S. (1-Object) vs. C.S. (1-Call-Site)



CG: { [ ]:4 → [o_3]:C.m(),
[o_3]:14 → [o_3]:C.id(Number),
[o_3]:15 → [o_3]:C.id(Number),
[o_3]:16 → [o_{12}]:One.get(),
[o_3]:16 → [o_{13}]:Two.get() }

Final results



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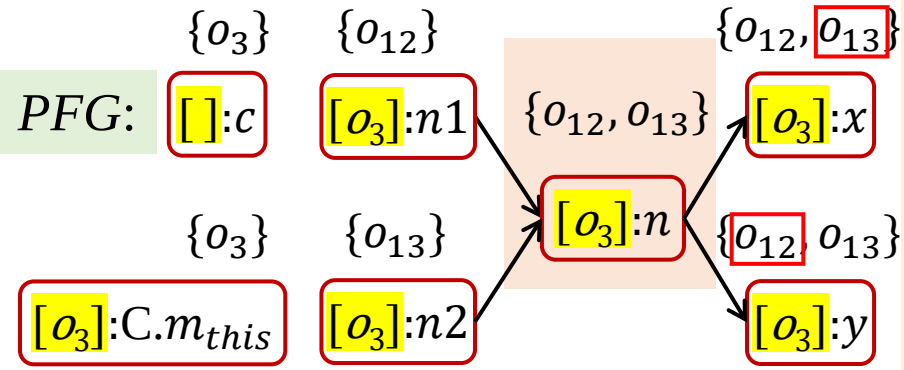
C.S. (1-Object) vs. C.S. (1-Call-Site)



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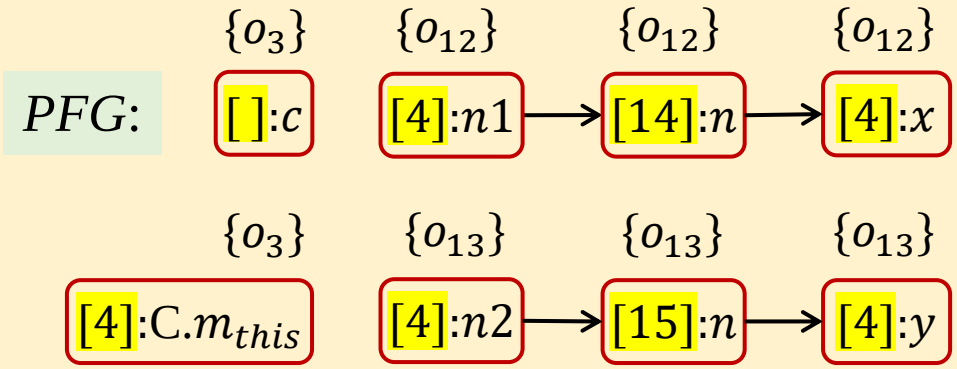
Spurious

Final results



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Call-Site vs. Object Sensitivity

- In theory, their precision is incomparable

Call-Site vs. Object Sensitivity

- In theory, their precision is incomparable
- In practice, object sensitivity generally outperforms call-site sensitivity for OO languages (like Java)

Call-Site vs. Object Sensitivity

	Time (s)		#may-fail-cast		#call-graph-edge	
	2-call	2-obj	2-call	2-obj	2-call	2-obj
batik	6,886	3,300	2,452	1,606	94,211	76,807
checkstyle	2,277	2,003	863	581	54,171	48,809
sunflow	5,570	1,208	2,504	1,837	100,701	89,866
findbugs	3,812	2,661	2,056	1,409	72,118	65,836
jpc	3,343	559	1,855	1,392	89,677	81,030
eclipse	1,896	146	886	546	42,872	38,151
chart	2,705	282	1,481	883	59,691	52,374
fop	5,503	1,200	1,975	1,446	79,524	71,408
xalan	1,927	1,093	919	533	48,763	44,871
bloat	5,712	3,525	1,699	1,193	58,696	53,143

For all numbers, lower is better (in terms of efficiency or precision)

Yue Li, Tian Tan, Anders Møller, and Yannis Smaragdakis. "*A Principled Approach to Selective Context Sensitivity for Pointer Analysis*". TOPLAS 2020.

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```
A a = (A) o;
```

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In general
- Precision: object > call-site
- Efficiency: object > call-site

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Type Sensitivity*

- Each context consists of a list of types
  - At a method call, use the type containing the allocation site of the receiver object with its heap context as callee context
  - A coarser abstraction over object sensitivity

$$\text{Select}(\_, \_, \boldsymbol{c'}: \boldsymbol{o_i}) = [t', \dots, t'', \text{InType}(o_i)]$$
$$\text{where } c' = [t', \dots, t'']$$

* Yannis Smaragdakis, Martin Bravenboer, and Ondrej Lhoták. "*Pick Your Contexts Well: Understanding Object-Sensitivity*". POPL 2011.

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```
1 class X {
2   void m() {
3     Y y = new Y();
4   }
5 }
```

$\text{InType}(o_3) = \text{X}$ (**not** Y!)

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Type vs. Object Sensitivity

Under the same *k*-limiting, the precision of type sensitivity is no better than object sensitivity

- Type sensitivity is a coarser abstraction over object sensitivity

```
1  class X {
2    void main() {
3      Y y1 = new Y();
4      y1.foo();
5      Y y2 = new Y();
6      y2.foo();
7      Y y3 = new Y();
8      y3.foo();
9    }
10 }
11 class Y {
12   void foo() {}
13 }
```

Contexts of `foo()`

- Object-sensitivity: ?

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9    }
10 }
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```

Contexts of `foo()`

- Object-sensitivity: $[o_3], [o_5], [o_7]$
- Type-sensitivity: ?

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7      Y y3 = new Y();
8      y3.foo();
9    }
10 }
11 class Y {
12   void foo() {}
13 }
```

Compared to object sensitivity, type sensitivity trades precision for better efficiency, by merging the allocation sites in the same type in contexts

- In practice, type sensitivity is less precise but more efficient than object sensitivity

Contexts of `foo()`

- Object-sensitivity: $[o_3], [o_5], [o_7]$
- Type-sensitivity: $[\mathrm{X}]$

Call-Site vs. Object vs. Type Sensitivity

	Time (s)			#may-fail-cast			#call-graph-edge		
	2-call	2-obj	2-type	2-call	2-obj	2-type	2-call	2-obj	2-type
batik	6,886	3,300	378	2,452	1,606	1,938	94,211	76,807	77,337
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sunflow	5,570	1,208	197	2,504	1,837	2,247	100,701	89,866	90,967
findbugs	3,812	2,661	265	2,056	1,409	1,683	72,118	65,836	66,443
jpc	3,343	559	128	1,855	1,392	1,599	89,677	81,030	81,527
eclipse	1,896	146	57	886	546	665	42,872	38,151	38,337
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The X You Need To Understand in This Lecture

- Algorithm for context-sensitive pointer analysis
- Common context sensitivity variants
- Differences and relationship among common context sensitivity variants

软件分析

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计算机科学与技术系
程序设计语言与静态分析研究组
李樾 谭添