软件分析

南京大学
计算机科学与技术系
程序设计语言与
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Static Program Analysis

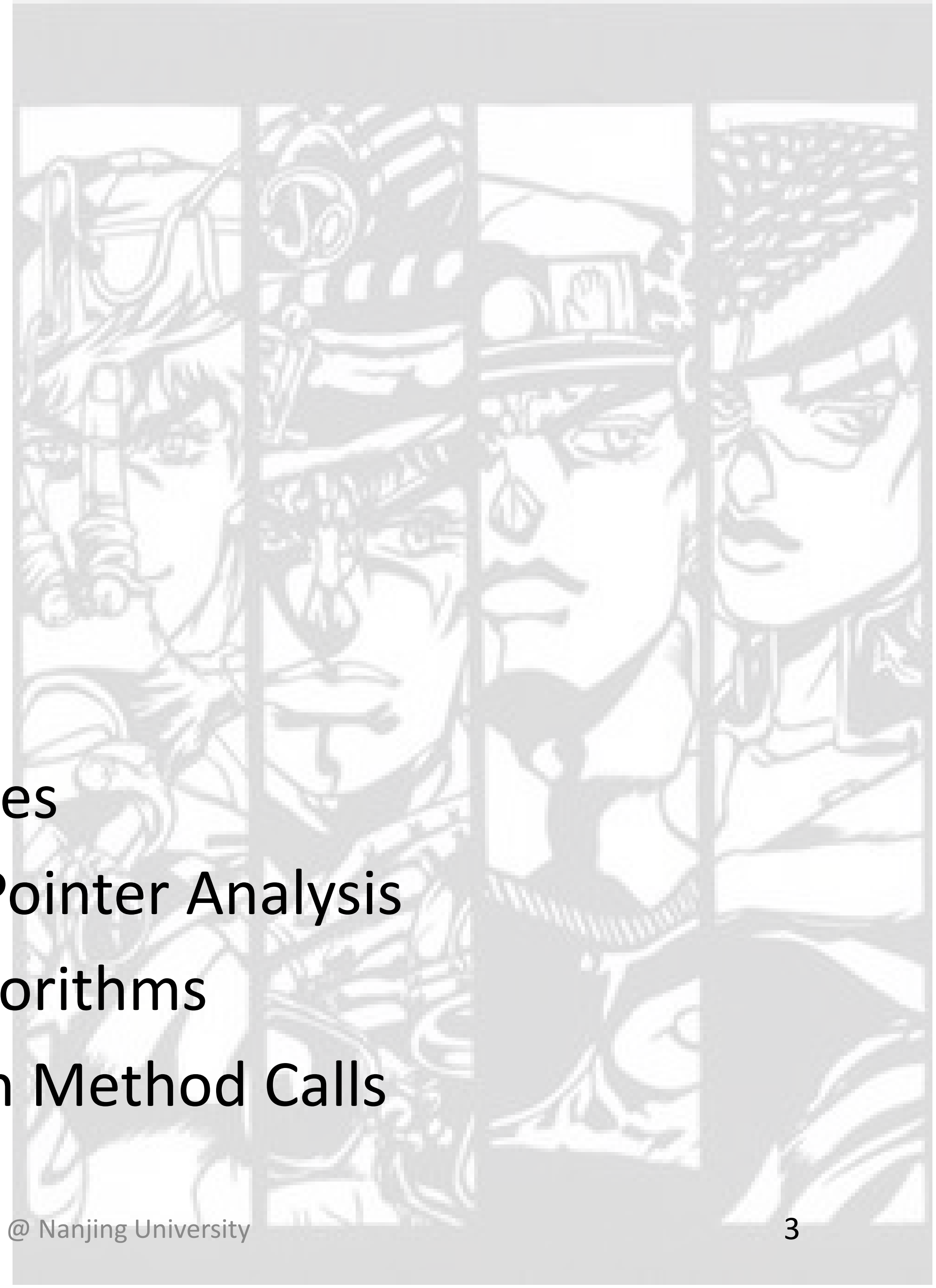
Pointer Analysis Foundations (I)

Nanjing University

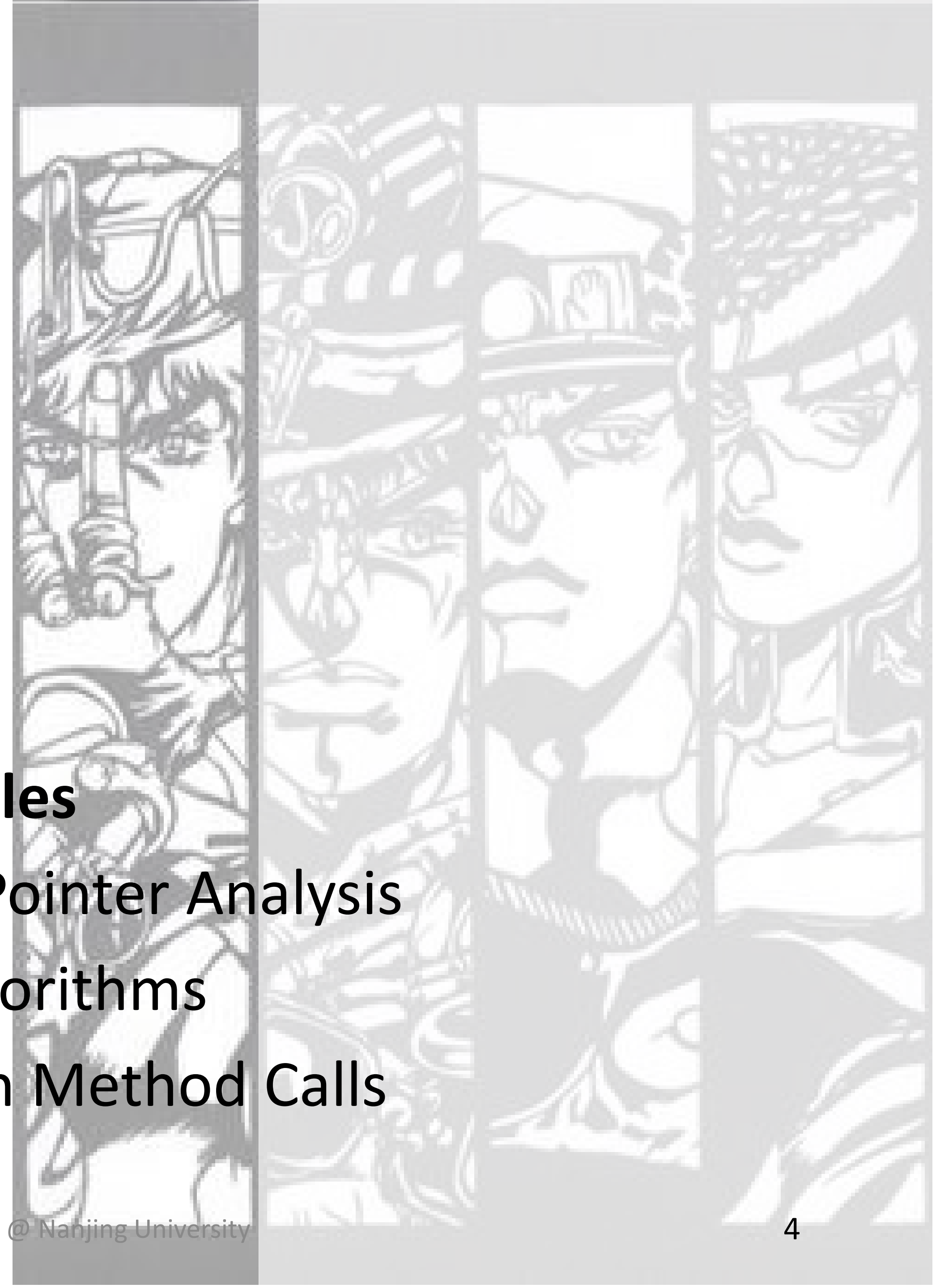
Tian Tan

2021

Contents



Contents



1. **Pointer Analysis: Rules**
2. How to Implement Pointer Analysis
3. Pointer Analysis: Algorithms
4. Pointer Analysis with Method Calls

Pointer-Affecting Statements

New	`x = new T()`
Assign	`x = y`
Store	`x.f = y`
Load	`y = x.f`
Call	`r = x.k(a, …)`

First focus on these statements
(suppose the program has just one method)

Will come back to this in
pointer analysis with method calls

Domain and Notations

Variables:	$x, y \in \mathrm{V}$
Fields:	$f, g \in \mathrm{F}$
Objects:	$o_i, o_j \in \mathrm{O}$
Instance fields:	$o_i.f, o_j.g \in \mathrm{O} \times \mathrm{F}$
Pointers:	$\mathrm{Pointer} = \mathrm{V} \cup (\mathrm{O} \times \mathrm{F})$
Points-to relations:	$pt : \mathrm{Pointer} \longrightarrow \mathcal{P}(\mathrm{O})$

- $\mathcal{P}(\mathrm{O})$ denotes the powerset of O
- $pt(p)$ denotes the points-to set of p

Rules

Kind	Statement	Rule
New	`i: x = new T()`	$\dfrac{}{o_i \in pt(x)}$
Assign	`x = y`	$\dfrac{o_i \in pt(y)}{o_i \in pt(x)}$
Store	`x.f = y`	$\dfrac{o_i \in pt(x),\ o_j \in pt(y)}{o_j \in pt(o_i.f)}$
Load	`y = x.f`	$\dfrac{o_i \in pt(x),\ o_j \in pt(o_i.f)}{o_j \in pt(y)}$

Rules

Kind	Statement	Rule	
New	`i: x = new T()`	$\dfrac{}{o_i \in pt(x)}$	← unconditional
Assign	`x = y`	$\dfrac{o_i \in pt(y)}{o_i \in pt(x)}$	← premises ← conclusion
Store	`x.f = y`	$\dfrac{o_i \in pt(x),\ \ o_j \in pt(y)}{o_j \in pt(o_i.f)}$	
Load	`y = x.f`	$\dfrac{o_i \in pt(x),\ \ o_j \in pt(o_i.f)}{o_j \in pt(y)}$	

Rule: New

$$\frac{}{o_i \in pt(x)}$$

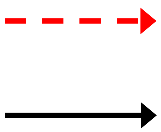
Conclusion

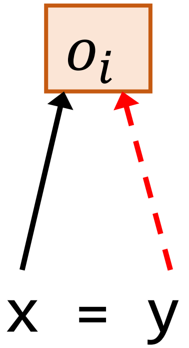
$\boxed{o_i}$

i: `x = new T()`

Rule: Assign

$$\frac{o_i \in pt(y)}{o_i \in pt(x)}$$


Premises
Conclusion

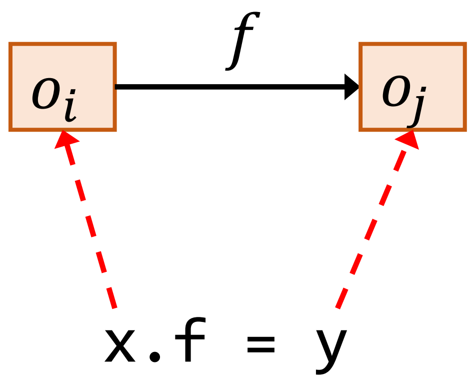


Rule: Store

$$\frac{o_i \in pt(x),\ o_j \in pt(y)}{o_j \in pt(o_i.f)}$$

Premises

Conclusion



Rule: Load

$$\frac{o_i \in pt(x),\ o_j \in pt(o_i.f)}{o_j \in pt(y)}$$

Premises

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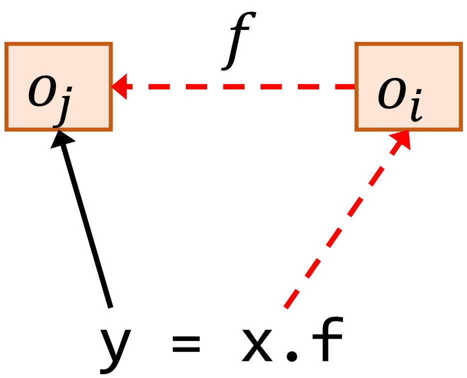
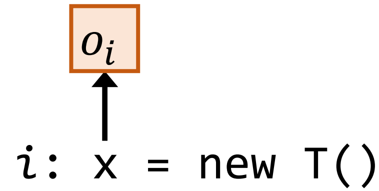
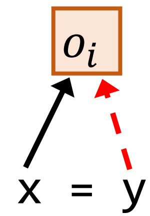
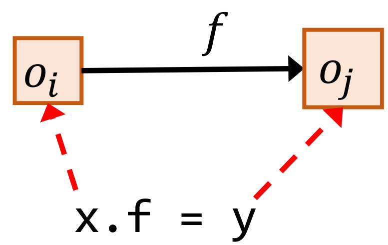
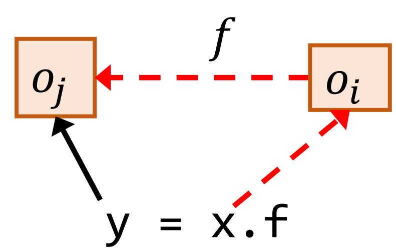


Rules

Premises

Conclusion

Kind	Rule	Illustration
New	$\dfrac{}{o_i \in pt(x)}$	
Assign	$\dfrac{o_i \in pt(y)}{o_i \in pt(x)}$	
Store	$\dfrac{o_i \in pt(x),\ \ o_j \in pt(y)}{o_j \in pt(o_i.f)}$	
Load	$\dfrac{o_i \in pt(x),\ \ o_j \in pt(o_i.f)}{o_j \in pt(y)}$	

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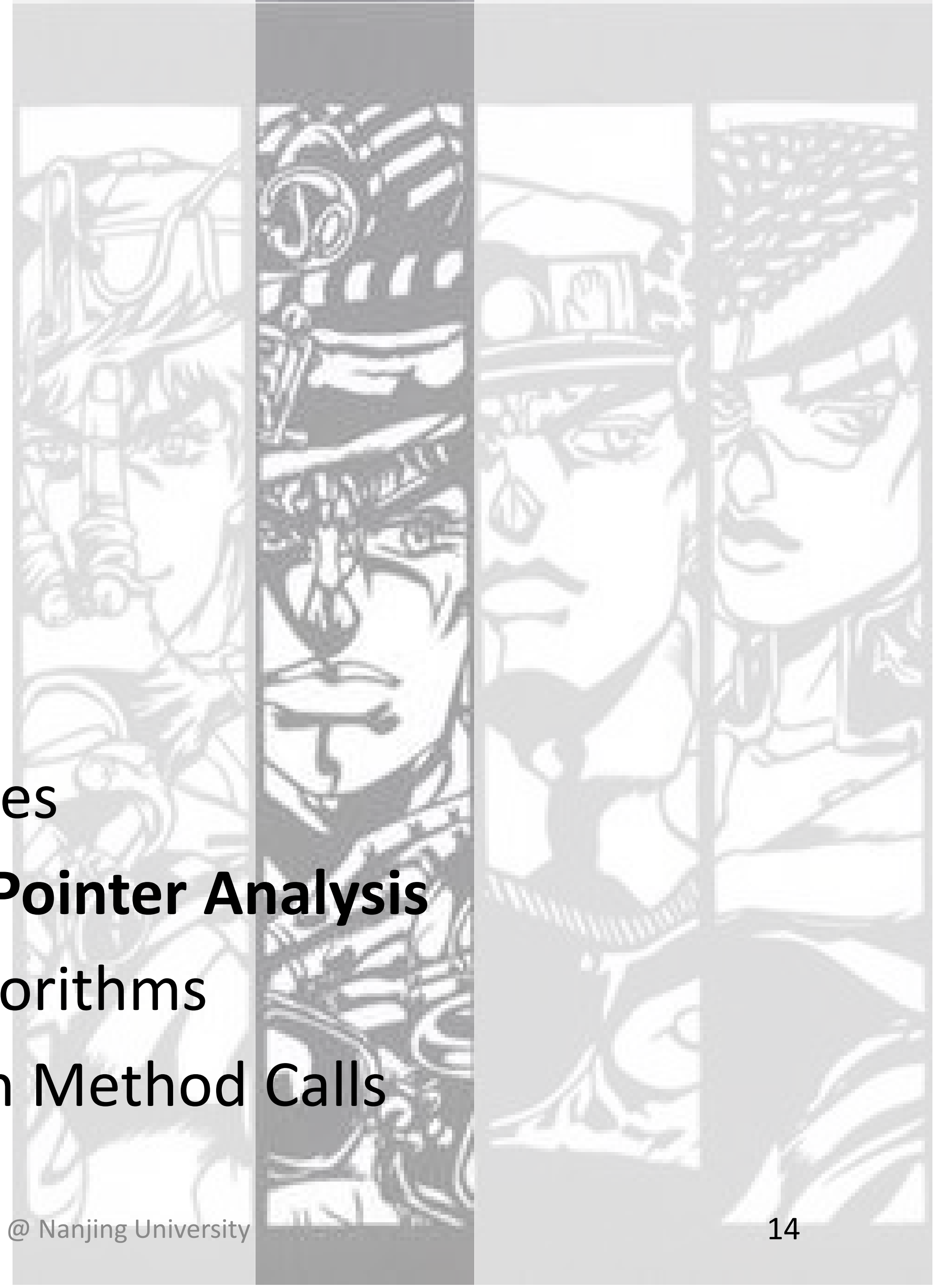

1. Pointer Analysis: Rules
2. **How to Implement Pointer Analysis**
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Our Pointer Analysis Algorithms

- A complete whole-program pointer analysis
- Carefully designed for understandability
- Easy to follow and implement

How to Implement Pointer Analysis?

- Essentially, pointer analysis is to **propagate** points-to information among pointers (variables & fields)

Kind	Statement	Rule
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$
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Pointer analysis as solving a system of inclusion constraints for pointers

Referred as *Andersen-style analysis**

* Lars Ole Andersen, 1994. "*Program Analysis and Specialization for the C Programming Language*". Ph.D. Thesis. University of Copenhagen.

How to Implement Pointer Analysis?

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Key to implementation: when $pt(x)$ is **changed**, **propagate** the **changed part** to the **related pointers** of x

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Load	`y = x.f`	$\frac{o_i \in pt(x),\ o_j \in pt(\boldsymbol{o_i.f})}{o_j \in pt(\boldsymbol{y})}$

Solution

- We use a **graph** to connect related pointers
- When $pt(x)$ changes, propagate the changed part to x's **successors**

Key to implementation: when $pt(x)$ is **changed**, **propagate** the **changed part** to the **related pointers** of x

Pointer Flow Graph (PFG)

Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program.

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Pointer flow graph of a program is a *directed graph* that expresses how objects flow among the pointers in the program.

- Nodes: $\text{Pointer} = \text{V} \cup (\text{O} \times \text{F})$

  A node n represents *a variable* or *a field of an abstract object*

- Edges: $\text{Pointer} \times \text{Pointer}$

  An edge $x \rightarrow y$ means that the objects pointed by pointer x *may flow to* (and also be pointed to by) pointer y

Pointer Flow Graph: Edges

- PFG edges are added according to the statements of the program and the corresponding rules

Kind	Statement	Rule
New	`i: x = new T()`	$\frac{}{o_i \in pt(x)}$
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Pointer Flow Graph: Edges

- PFG edges are added according to the statements of the program and the corresponding rules

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\dfrac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$
Store	`x.f = y`	$\dfrac{o_i \in pt(x),\ \ o_j \in pt(\boldsymbol{y})}{o_j \in pt(\boldsymbol{o_i.f})}$	$o_i.f \leftarrow y$
Load	`y = x.f`	$\dfrac{o_i \in pt(x),\ \ o_j \in pt(\boldsymbol{o_i.f})}{o_j \in pt(\boldsymbol{y})}$	$y \leftarrow o_i.f$

Pointer Flow Graph: An Example

Program

$(o_i \in pt(c), o_i \in pt(d))$

```
a = b;      ①
c.f = a;    ②
d = c;      ③
c.f = d;    ④
e = d.f;    ⑤
```

Pointer flow graph

- Variable node v
- Instance field node o_i.f

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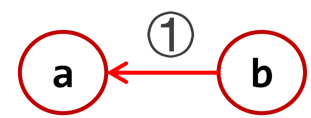

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```

?

Pointer flow graph

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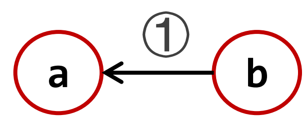

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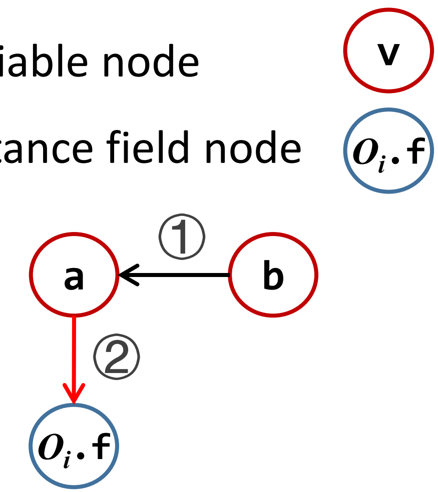

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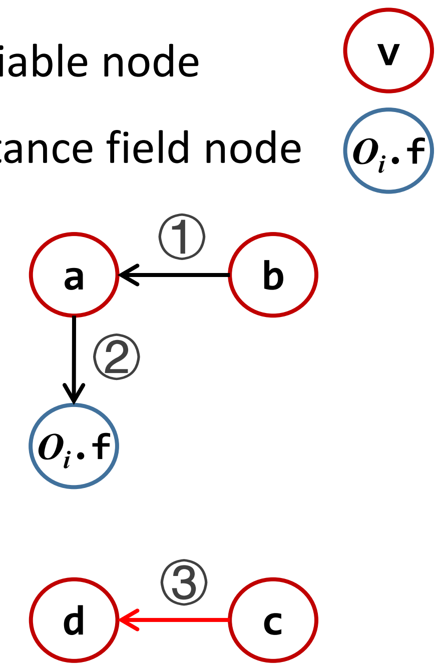

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Pointer flow graph

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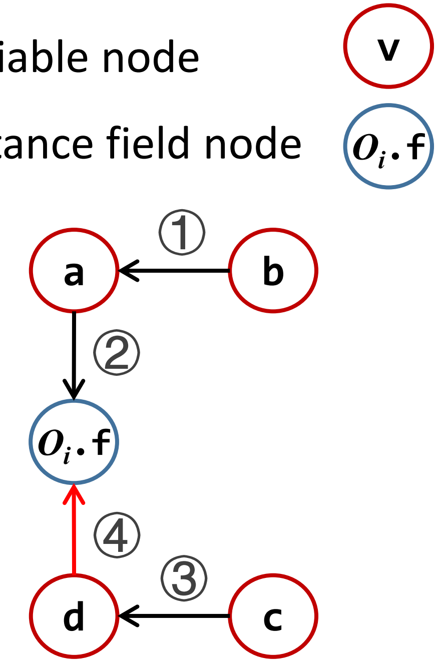

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Pointer flow graph

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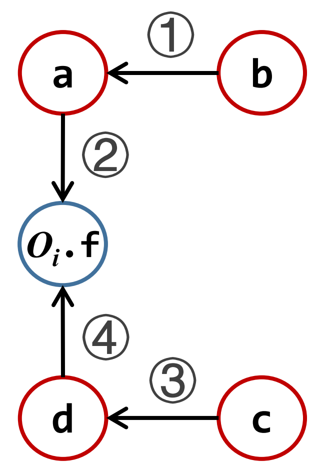

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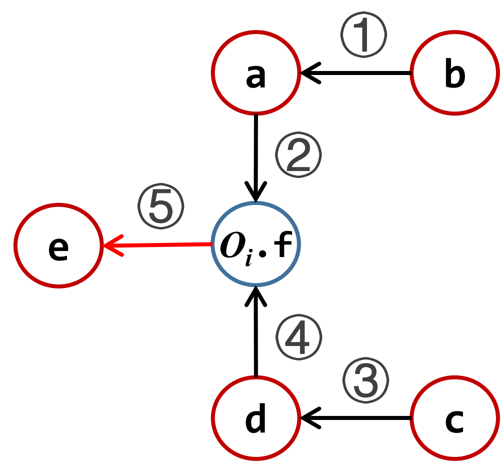

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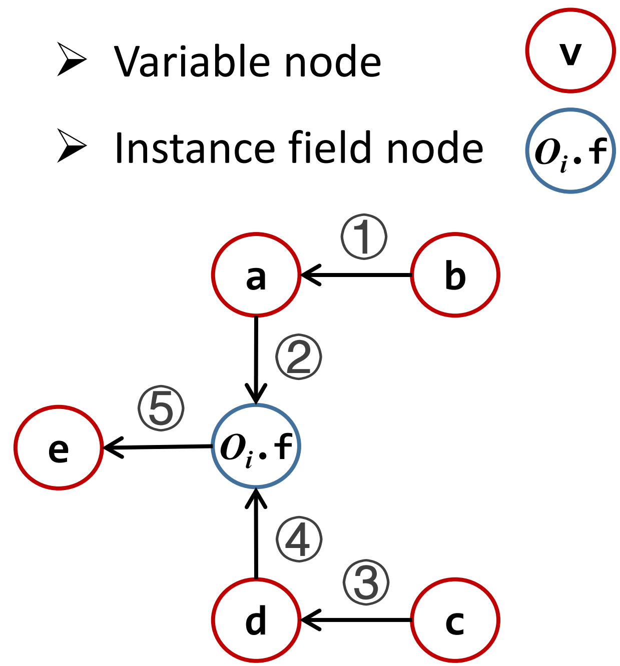

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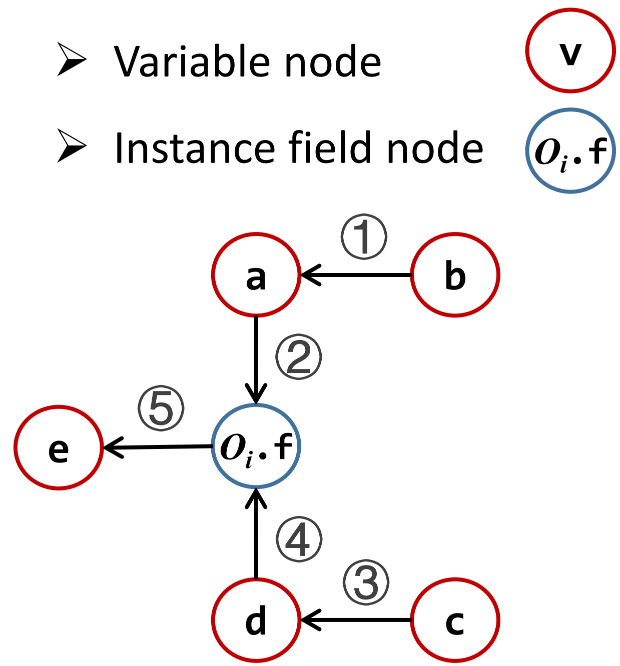

With PFG, pointer analysis can be solved by computing ***transitive closure*** of the PFG

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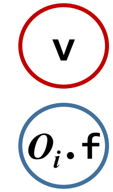

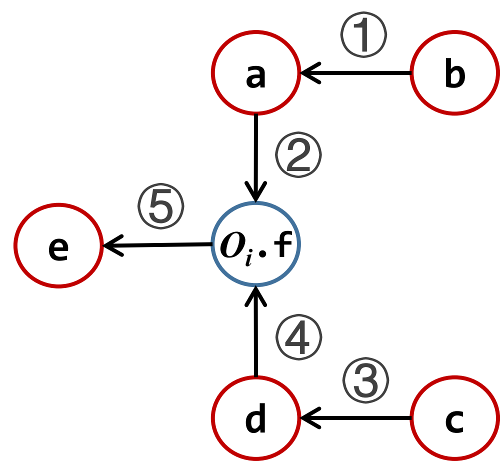

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E.g, **e** is reachable from **b** on the PFG, which means that the objects pointed by **b** may flow to and also be pointed by **e**

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Pointer flow graph

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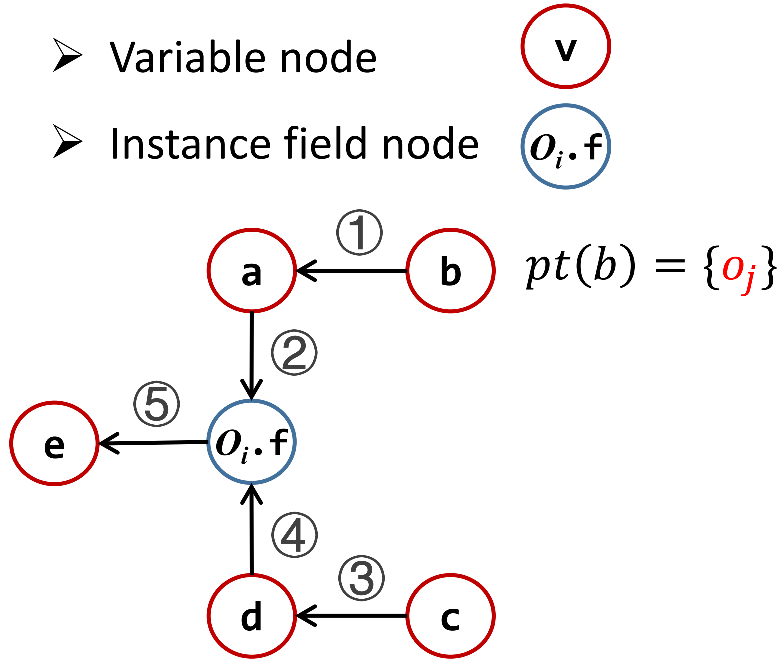

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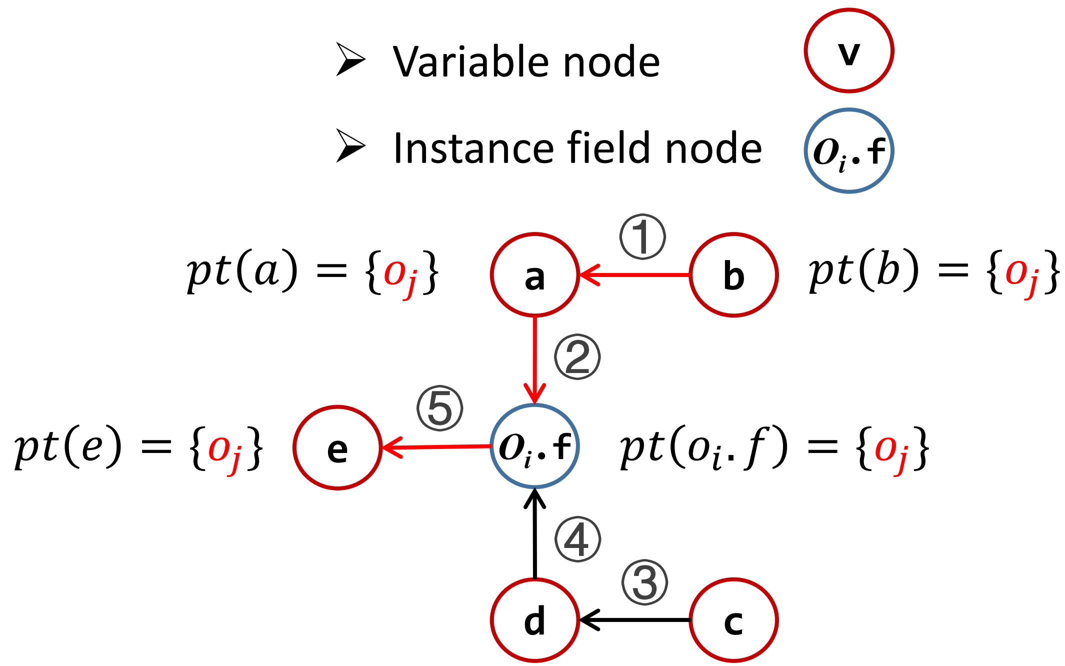

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Implementing Pointer Analysis

1. Build pointer flow graph (PFG)

2. Propagate points-to information on PFG

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Mutually dependent

2. Propagate points-to information on PFG

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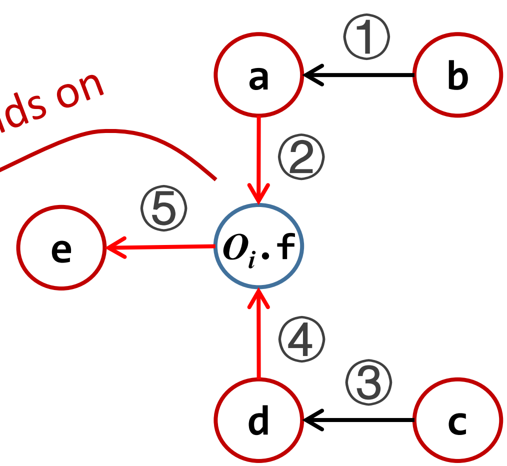
depends on

Pointer flow graph



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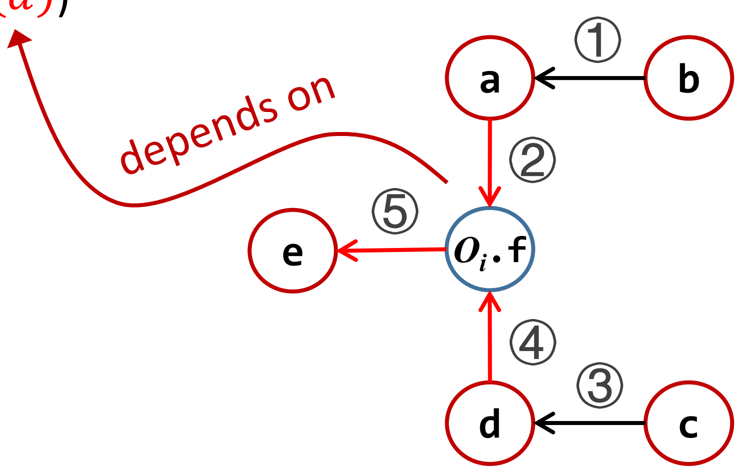
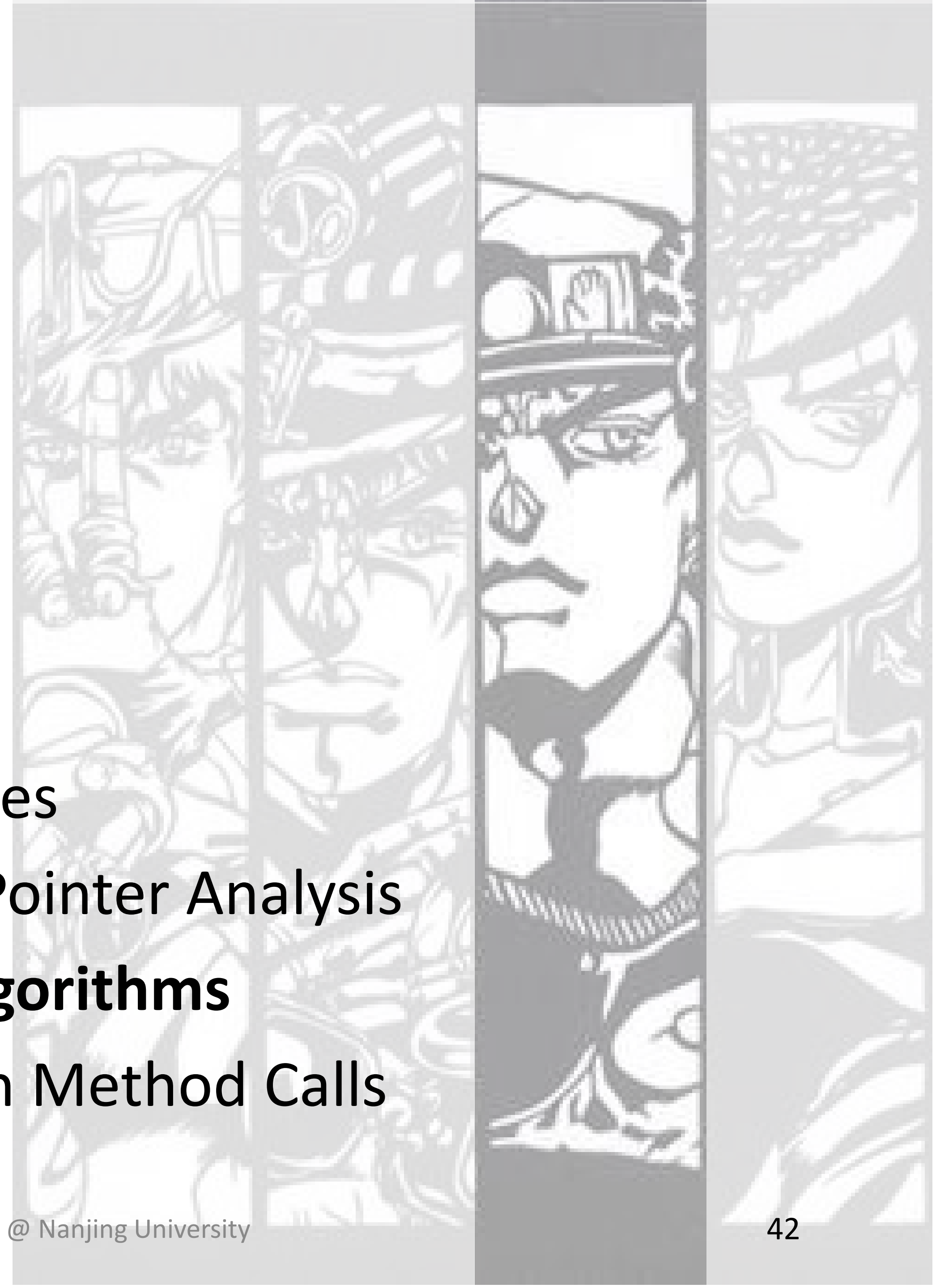


PFG is dynamically **updated** during pointer analysis

Contents



Pointer Analysis: Algorithms

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL
  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then**
    add $s \rightarrow t$ to PFG
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s)\rangle$ to WL

Propagate(n, pts)
  if pts is not empty **then**
    $pt(n)$ ⋃= pts
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts\rangle$ to WL

S	Set of statements of the input program
WL	Work list
PFG	Pointer flow graph

Pointer Analysis: Algorithms

Solve(S)
    $WL = [\,], PFG = \{\}$
    foreach `i: x = new T()` $\in S$ **do**
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    foreach `x = y` $\in S$ **do**
        AddEdge(y, x)

    while WL is not empty **do**
        remove $\langle n, pts\rangle$ from WL
        $\Delta = pts - pt(n)$
        Propagate(n, Δ)
        if n represents a variable `x` **then**
            foreach $o_i \in \Delta$ **do**
                foreach `x.f = y` $\in S$ **do**
                    AddEdge($y, o_i.f$)
                foreach `y = x.f` $\in S$ **do**
                    AddEdge($o_i.f, y$)

main algorithm

AddEdge(s, t)
    if $s \rightarrow t \notin PFG$ **then**
        add $s \rightarrow t$ to PFG
        if $pt(s)$ is not empty **then**
            add $\langle t, pt(s)\rangle$ to WL

Propagate(n, pts)
    if pts is not empty **then**
        $pt(n) \mathrel{\cup=} pts$
        foreach $n \rightarrow s \in PFG$ **do**
            add $\langle s, pts\rangle$ to WL

S	Set of statements of the input program
WL	Work list
PFG	Pointer flow graph

Pointer Analysis: Algorithms

```
Solve(S)
  WL = [ ], PFG = {}
  foreach i: x = new T() ∈ S do
    add ⟨x, {o_i}⟩ to WL

  foreach x = y ∈ S do
    AddEdge(y, x)

  while WL is not empty do
    remove ⟨n, pts⟩ from WL
    Δ = pts – pt(n)
    Propagate(n, Δ)
    if n represents a variable x then
      foreach o_i ∈ Δ do
        foreach x.f = y ∈ S do
          AddEdge(y, o_i.f)
        foreach y = x.f ∈ S do
          AddEdge(o_i.f, y)
```

main algorithm

```
AddEdge(s, t)
  if s → t ∉ PFG then
    add s → t to PFG
    if pt(s) is not empty then
      add ⟨t, pt(s)⟩ to WL

Propagate(n, pts)
  if pts is not empty then
    pt(n) ⋃= pts
    foreach n → s ∈ PFG do
      add ⟨s, pts⟩ to WL
```

S	Set of statements of the input program
WL	Work list
PFG	Pointer flow graph

Worklist (WL)

- Worklist contains the points-to information to be processed
  - $WL \subseteq \langle \text{Pointer}, \mathcal{P}(\text{O}) \rangle^*$

- Each worklist entry $\langle n, pts \rangle$ is a pair of pointer n and points-to set pts, which means that pts should be propagated to $pt(n)$
  - E.g., $\left[\langle x, \{o_i\} \rangle, \langle y, \{o_j, o_k\} \rangle, \langle o_j.f, \{o_l\} \rangle \ldots \right]$

Main Algorithm

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

Initialize the analysis

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\frac{}{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\frac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

```
Solve(S)
    WL = [ ], PFG = {}
    foreach i: x = new T() ∈ S do
        add ⟨x, {o_i}⟩ to WL
    foreach x = y ∈ S do
        AddEdge(y, x)

    while WL is not empty do
        remove ⟨n, pts⟩ from WL
        Δ = pts – pt(n)
        Propagate(n, Δ)
```

Initialize the analysis

Add assign edges to PFG

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\dfrac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then** ①
    add $s \rightarrow t$ to PFG
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s)\rangle$ to WL

① Do nothing if $s \rightarrow t$ is already in PFG

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\dfrac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then** ①
    add $s \rightarrow t$ to PFG ②
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s)\rangle$ to WL

① Do nothing if $s \rightarrow t$ is already in PFG

② **Add PFG edge**

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\frac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` ∈ S **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` ∈ S **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then** ①
    add $s \rightarrow t$ to PFG ②
    if $pt(s)$ is not empty **then** ③
      add $\langle t, pt(s)\rangle$ to WL

① Do nothing if $s \rightarrow t$ is already in PFG

② **Add PFG edge**

③ **Ensure** every object pointed by s is also pointed by t

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\dfrac{o_i \in pt(y)}{o_i \in pt(x)}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

Process the entries in WL

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\frac{}{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\frac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
    WL = [ ], PFG = {}
    foreach `i: x = new T()` $\in S$ **do**
        add $\langle x, \{o_i\}\rangle$ to WL

    foreach `x = y` $\in S$ **do**
        AddEdge(y, x)

    while WL is not empty **do**
        remove $\langle n, pts\rangle$ from WL
        $\Delta = pts - pt(n)$
        Propagate(n, Δ)

Process the entries in WL

remove $\langle x, \{o_1, o_3\}\rangle$ from WL

$pt(x) = \{o_1, o_2\}$

$$\Delta = pts - pt(x) = \{o_1, o_3\} - \{o_1, o_2\} = \{o_3\}$$

Propagate(x, $\{o_3\}$)

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\frac{}{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\frac{o_i \in pt(\mathbf{y})}{o_i \in pt(\mathbf{x})}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

Propagate(n, pts)
  if pts is not empty **then** ①
    $pt(n) \cup= pts$
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts\rangle$ to WL

① Do nothing if pts is empty

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\frac{}{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\frac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

$\text{Solve}(S)$
  $WL = [\,], PFG = \{\}$
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL
  foreach `x = y` $\in S$ **do**
    $\text{AddEdge}(y, x)$

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    $\text{Propagate}(n, \Delta)$

$\text{Propagate}(n, pts)$
  if pts is not empty **then** ①
    $pt(n) \cup= pts$ ②
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts\rangle$ to WL

① Do nothing if pts is empty

② **Propagate pts to points-to set of n**

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\dfrac{}{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\dfrac{o_i \in pt(\boldsymbol{y})}{o_i \in pt(\boldsymbol{x})}$	$x \leftarrow y$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` ∈ S **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` ∈ S **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

Propagate(n, pts)
  if pts is not empty **then** ①
    $pt(n) \cup= pts$ ②
    foreach $n \rightarrow s \in PFG$ **do** ③
      add $\langle s, pts\rangle$ to WL

① Do nothing if pts is empty

② **Propagate pts to points-to set of n**

③ Propagate pts (**the changed part**) to n's successors on PFG

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\overline{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\frac{o_i \in pt(y)}{o_i \in pt(x)}$	$x \leftarrow y$

Differential Propagation

Solve(S)
…
while WL is not empty **do**
remove $\langle n, pts \rangle$ from WL
$\boxed{\begin{array}{l}\Delta = pts - pt(n) \\ \text{Propagate}(n, \Delta)\end{array}}$
…

Why?

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

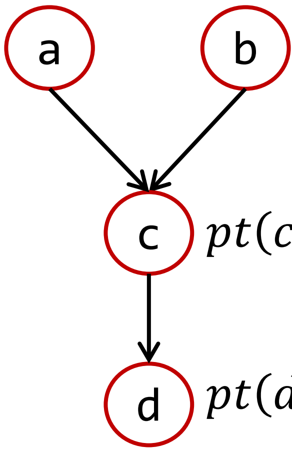
```
Solve(S)
  …
  while WL is not empty do
    remove ⟨n, pts⟩ from WL
    Δ = pts – pt(n)
    Propagate(n, Δ)
    …
```

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

Solve(S)
  …
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\boxed{\begin{array}{l}\Delta = pts - pt(n) \\ \text{Propagate}(n, \Delta)\end{array}}$
    …

$pt(a) = \{o_1, o_2, o_3\}$ $pt(b) = \{o_1, o_3, o_5\}$



$pt(c) = \{\}$

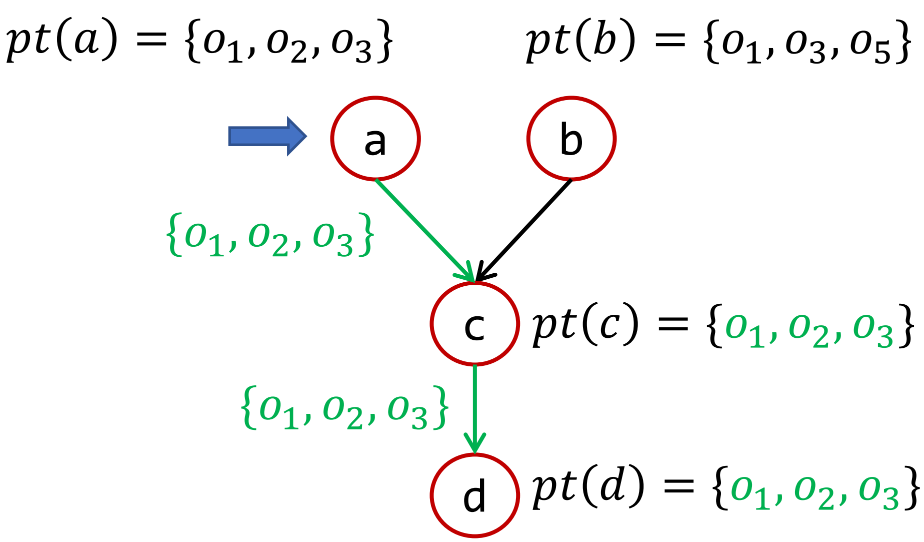
$pt(d) = \{\}$

PFG

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

Solve(S)
    …
    while WL is not empty **do**
        remove $\langle n, pts \rangle$ from WL
        $\boxed{\begin{array}{l}\Delta = pts - pt(n) \\ \text{Propagate}(n, \Delta)\end{array}}$
        …

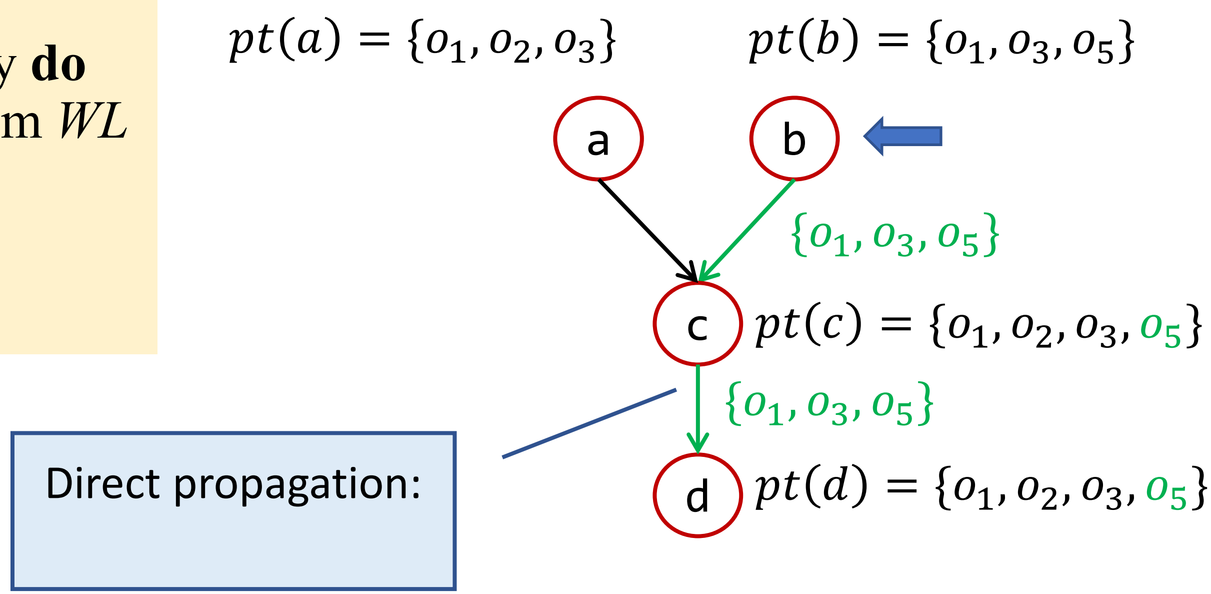


PFG

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

Solve(S)
    …
    while WL is not empty **do**
        remove $\langle n, pts \rangle$ from WL
        $\boxed{\Delta = pts - pt(n)}$
        $\boxed{\text{Propagate}(n, \Delta)}$
        …

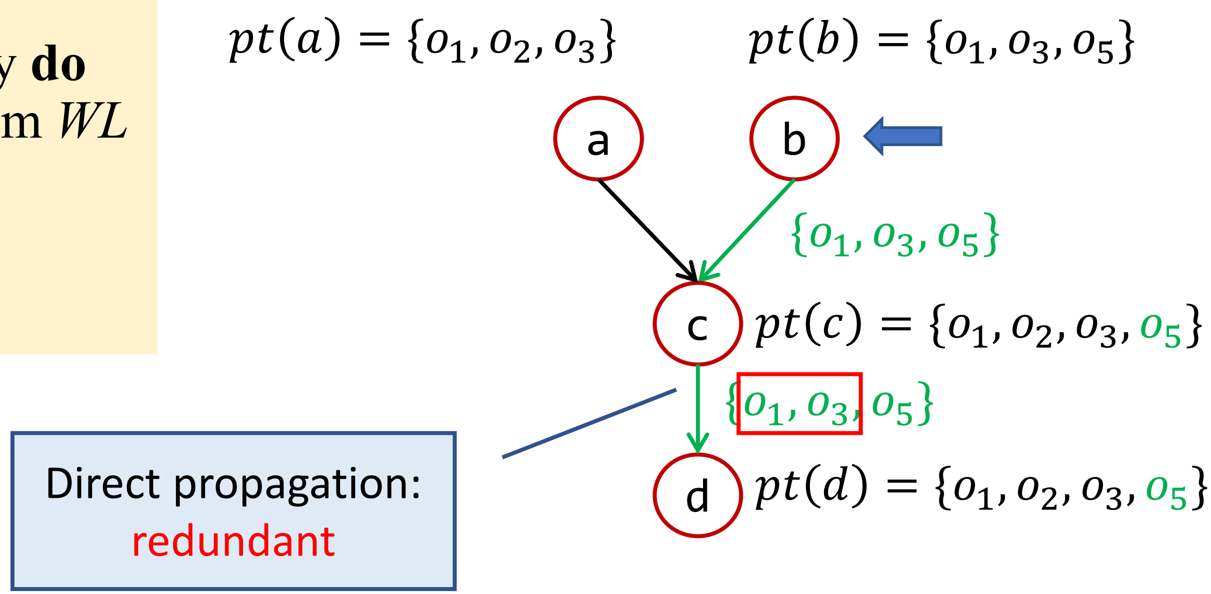


PFG

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

```
Solve(S)
    …
    while WL is not empty do
        remove ⟨n, pts⟩ from WL
        Δ = pts – pt(n)
        Propagate(n, Δ)
        …
```

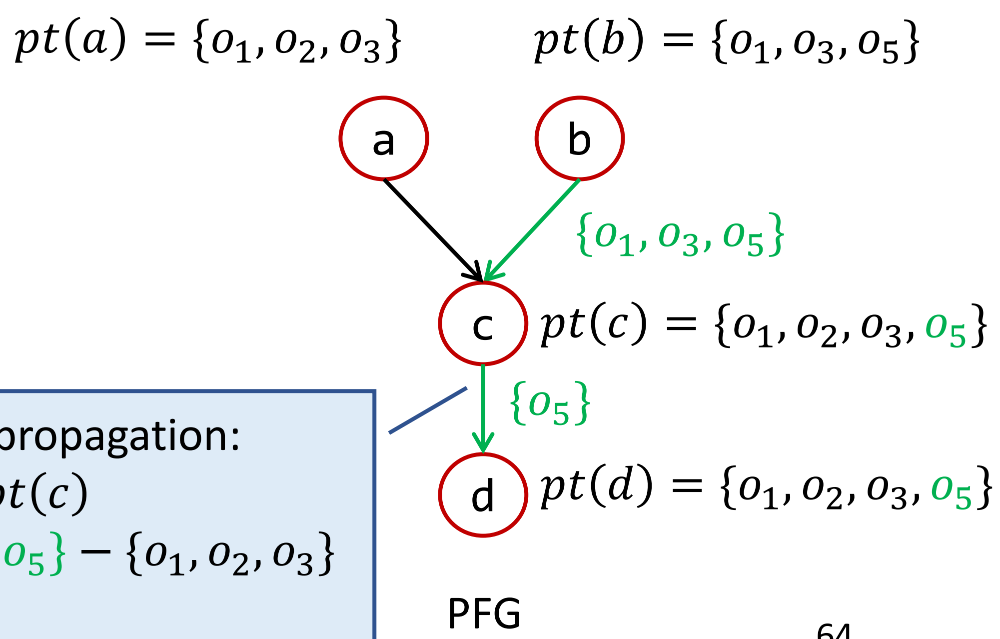


PFG

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

Solve(S)
    …
    while WL is not empty **do**
        remove $\langle n, pts \rangle$ from WL
        $\boxed{\begin{array}{l}\Delta = pts - pt(n) \\ \text{Propagate}(n, \Delta)\end{array}}$
        …



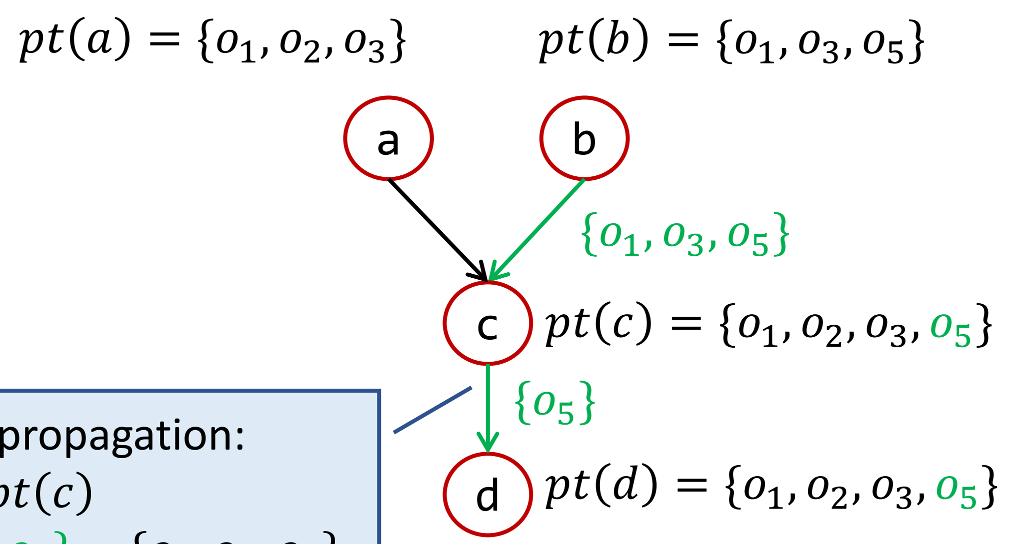
In practice, Δ is usually small compared with the original set, so propagating only the new points-to information (Δ) improves efficiency

Differential propagation:
$$\Delta = pts - pt(c) = \{o_1, o_3, o_5\} - \{o_1, o_2, o_3\} = \{o_5\}$$

Differential Propagation

- ***Differential propagation*** is employed to avoid propagation and processing of redundant points-to information
- **Insight**: existing points-to information in $pt(n)$ have already been propagated to n's successors, and ***no need*** to be propagated again

Solve(S)
  …
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\boxed{\Delta = pts - pt(n)}$
    $\boxed{\text{Propagate}(n, \Delta)}$
  …



PFG

Besides, Δ is also important for efficiency when handling stores, loads, and method calls, as explained later

Differential propagation:
$$\begin{aligned}\Delta &= pts - pt(c) \\ &= \{o_1, o_3, o_5\} - \{o_1, o_2, o_3\} \\ &= \{o_5\}\end{aligned}$$

Handling of New and Assign

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` ∈ S **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` ∈ S **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)

Kind	Statement	Rule	PFG Edge
New	`i: x = new T()`	$\dfrac{}{o_i \in pt(x)}$	N/A
Assign	`x = y`	$\dfrac{o_i \in pt(\mathbf{y})}{o_i \in pt(\mathbf{x})}$	$x \leftarrow y$

Handling of Store and Load

Solve(S)
  …
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)

Kind	Statement	Rule	PFG Edge
Store	`x.f = y`	$\dfrac{o_i \in pt(\boldsymbol{x}),\ o_j \in pt(\boldsymbol{y})}{o_j \in pt(\boldsymbol{o_i.f})}$	$o_i.f \leftarrow y$
Load	`y = x.f`	$\dfrac{o_i \in pt(\boldsymbol{x}),\ o_j \in pt(\boldsymbol{o_i.f})}{o_j \in pt(\boldsymbol{y})}$	$y \leftarrow o_i.f$

Handling of Store and Load

Solve(S)
    …
    while WL is not empty **do**
        remove $\langle n, pts\rangle$ from WL
        $\Delta = pts - pt(n)$
        Propagate(n, Δ)
        if n represents a variable `x` **then**
            foreach $o_i \in \Delta$ **do**
                foreach `x.f = y` $\in S$ **do**
                    AddEdge(y, $o_i.f$)
                foreach `y = x.f` $\in S$ **do**
                    AddEdge($o_i.f$, y)

New points-to information may introduce new PFG edges

AddEdge(s, t)
    if $s \rightarrow t \notin PFG$ **then**
        add $s \rightarrow t$ to PFG
        if $pt(s)$ is not empty **then**
            add $\langle t, pt(s)\rangle$ to WL

Kind	Statement	Rule	PFG Edge
Store	`x.f = y`	$\dfrac{o_i \in pt(\boldsymbol{x}),\ o_j \in pt(\boldsymbol{y})}{o_j \in pt(\boldsymbol{o_i.f})}$	$o_i.f \leftarrow y$
Load	`y = x.f`	$\dfrac{o_i \in pt(\boldsymbol{x}),\ o_j \in pt(\boldsymbol{o_i.f})}{o_j \in pt(\boldsymbol{y})}$	$y \leftarrow o_i.f$

Algorithms: Review

```
Solve(S)
  WL = [ ], PFG = {}
  foreach i: x = new T() ∈ S do
    add ⟨x, {o_i}⟩ to WL

  foreach x = y ∈ S do
    AddEdge(y, x)

  while WL is not empty do
    remove ⟨n, pts⟩ from WL
    Δ = pts – pt(n)
    Propagate(n, Δ)
    if n represents a variable x then
      foreach o_i ∈ Δ do
        foreach x.f = y ∈ S do
          AddEdge(y, o_i.f)
        foreach y = x.f ∈ S do
          AddEdge(o_i.f, y)
```

main algorithm

```
AddEdge(s, t)
  if s → t ∉ PFG then
    add s → t to PFG
    if pt(s) is not empty then
      add ⟨t, pt(s)⟩ to WL

Propagate(n, pts)
  if pts is not empty then
    pt(n) ⋃= pts
    foreach n → s ∈ PFG do
      add ⟨s, pts⟩ to WL
```

S	Set of statements of the input program
WL	Work list
PFG	Pointer flow graph

An Example

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)

S:

```
b = new C();
a = b;
c = new C();
c.f = a;
d = c;
c.f = d;
e = d.f;
```

WL: [ ]

PFG:

An Example

```
Solve(S)
  WL = [ ], PFG = {}
  foreach i: x = new T() ∈ S do
    add ⟨x, {o_i}⟩ to WL

  foreach x = y ∈ S do
    AddEdge(y, x)

  while WL is not empty do
    remove ⟨n, pts⟩ from WL
    Δ = pts – pt(n)
    Propagate(n, Δ)
    if n represents a variable x then
      foreach o_i ∈ Δ do
        foreach x.f = y ∈ S do
          AddEdge(y, o_i.f)
        foreach y = x.f ∈ S do
          AddEdge(o_i.f, y)
```

S:

```
1 b = new C();
2 a = b;
3 c = new C();
4 c.f = a;
5 d = c;
6 c.f = d;
7 e = d.f;
```

WL: $[\langle b, \{o_1\}\rangle, \langle c, \{o_3\}\rangle]$

PFG:

An Example

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` ∈ S **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` ∈ S **do**
    AddEdge(y, x)

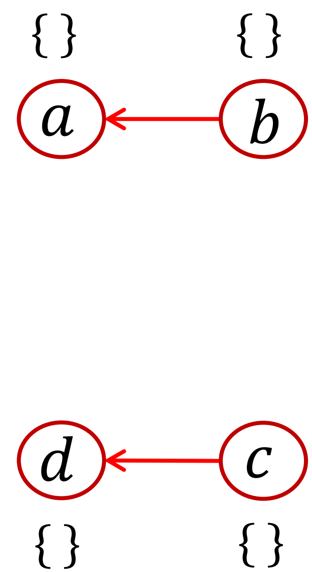
  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable x **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` ∈ S **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` ∈ S **do**
          AddEdge($o_i.f$, y)

S:

```
1 b = new C();
2 a = b;
3 c = new C();
4 c.f = a;
5 d = c;
6 c.f = d;
7 e = d.f;
```

WL: $[\langle b, \{o_1\}\rangle, \langle c, \{o_3\}\rangle]$

PFG:



An Example

Solve(S)
    …
    while WL is not empty **do**
        remove $\langle n, pts \rangle$ from WL
        $\Delta = pts - pt(n)$
        Propagate(n, Δ)
        if n represents a variable `x` **then**
            foreach $o_i \in \Delta$ **do**
                foreach `x.f = y` $\in S$ **do**
                    AddEdge(y, $o_i.f$)
                foreach `y = x.f` $\in S$ **do**
                    AddEdge($o_i.f$, y)

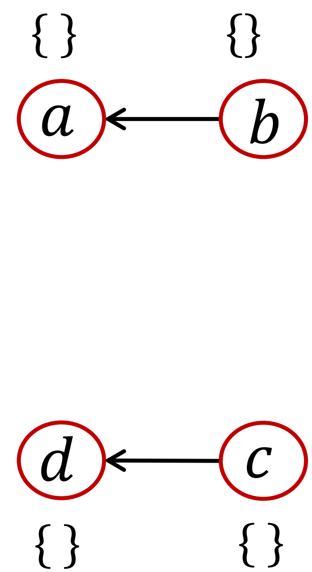
S:

```
b = new C();
a = b;
c = new C();
c.f = a;
d = c;
c.f = d;
e = d.f;
```

WL: $[\langle c, \{o_3\} \rangle]$

Processing: $\langle b, \{o_1\} \rangle$

PFG:



An Example

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Propagate(n, pts)
  if pts is not empty **then**
    $pt(n) \cup= pts$
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts\rangle$ to WL

S:

```
b = new C();
a = b;
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c.f = a;
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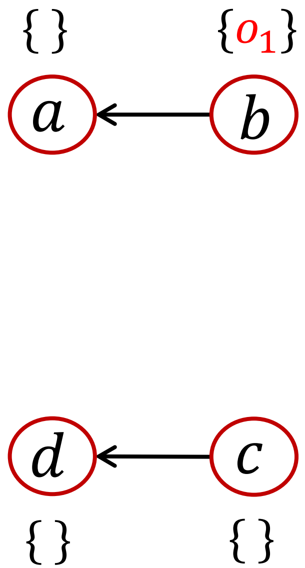
e = d.f;
```

WL: $[\langle c, \{o_3\}\rangle]$

Processing: $\langle b, \{o_1\}\rangle$

PFG:



An Example

Solve(S)

…

while WL is not empty **do**

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$\Delta = pts - pt(n)$

Propagate(n, Δ)

if n represents a variable x **then**

foreach $o_i \in \Delta$ **do**

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AddEdge($o_i.f$, y)

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if pts is not empty **then**

$pt(n)$ ∪= pts

foreach $n \rightarrow s \in PFG$ **do**

add $\langle s, pts \rangle$ to WL

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b = new C();
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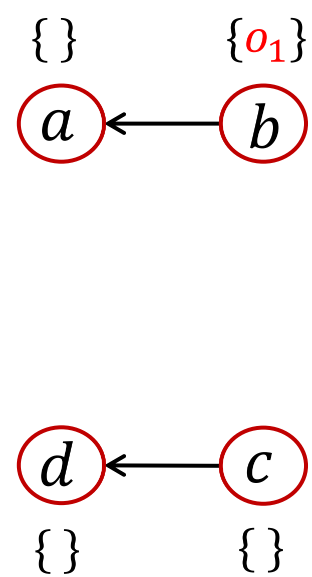
d = c;
c.f = d;
e = d.f;
```

WL: $[\langle c, \{o_3\}\rangle, \langle a, \{o_1\}\rangle]$

Processing: $\langle b, \{o_1\}\rangle$

PFG:



An Example

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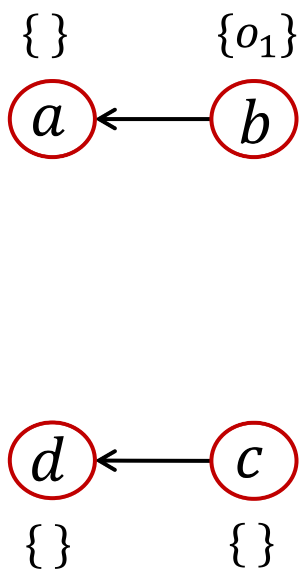
e = d.f;
```

WL: $[\langle a, \{o_1\} \rangle]$

Processing: $\langle c, \{o_3\} \rangle$

PFG:



An Example

Solve(S)

…

while WL is not empty **do**

remove $\langle n, pts \rangle$ from WL

$\Delta = pts - pt(n)$

Propagate(n, Δ)

if n represents a variable x **then**

foreach $o_i \in \Delta$ **do**

foreach `x.f = y` $\in S$ **do**

AddEdge($y, o_i.f$)

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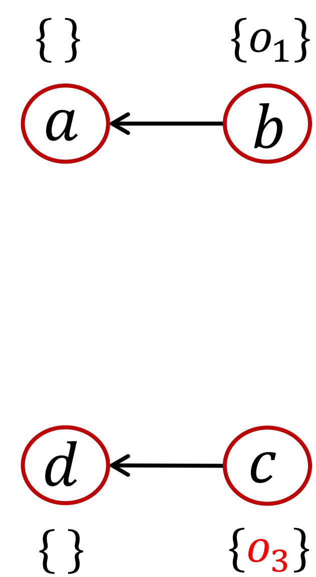
c.f = d;
e = d.f;
```

WL: $[\langle a, \{o_1\} \rangle]$

Processing: $\langle c, \{o_3\} \rangle$

PFG:



An Example

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  while WL is not empty **do**
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  if pts is not empty **then**
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    foreach $n \rightarrow s \in PFG$ **do**
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S:

```
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a = b;
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c.f = a;
d = c;
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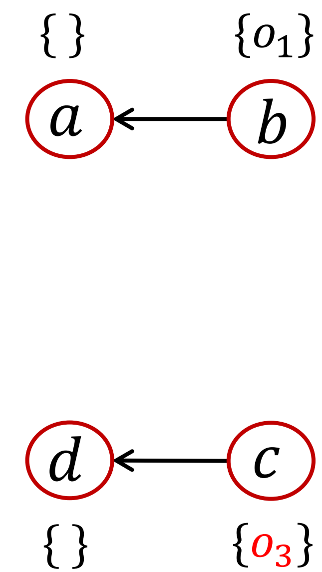
```

WL: $[\langle a, \{o_1\} \rangle, \langle d, \{o_3\} \rangle]$

Processing: $\langle c, \{o_3\} \rangle$

PFG:



An Example

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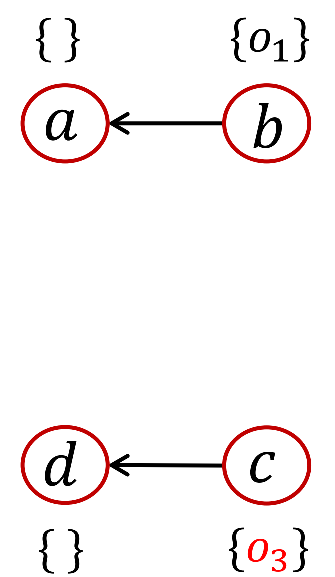
What next?

PFG:



An Example

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while WL is not empty **do**

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AddEdge(s, t)

if $s \rightarrow t \notin PFG$ **then**

add $s \rightarrow t$ to PFG

if $pt(s)$ is not empty **then**

add $\langle t, pt(s) \rangle$ to WL

S:

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b = new C();
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c.f = d;
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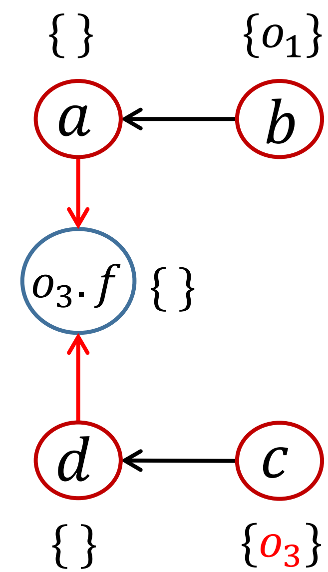
```

WL: $[\langle a, \{o_1\} \rangle, \langle d, \{o_3\} \rangle]$

Processing: $\langle c, \{o_3\} \rangle$

PFG:



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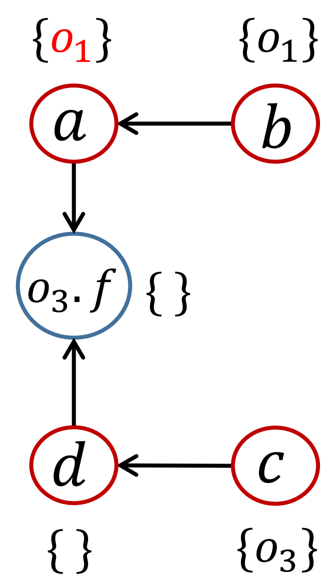
Processing: $\langle a, \{o_1\} \rangle$

PFG:



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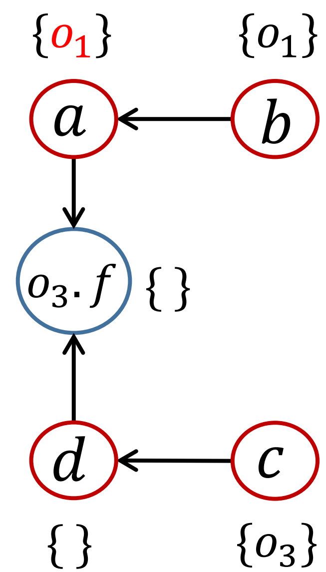
```

WL: $[\langle d, \{o_3\}\rangle, \langle o_3.f, \{o_1\}\rangle]$

Processing: $\langle a, \{o_1\}\rangle$

PFG:



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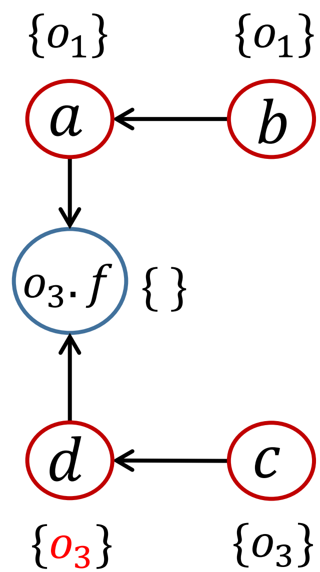
```

WL: $[\langle o_3.f, \{o_1\}\rangle]$

Processing: $\langle d, \{o_3\}\rangle$

PFG:



An Example

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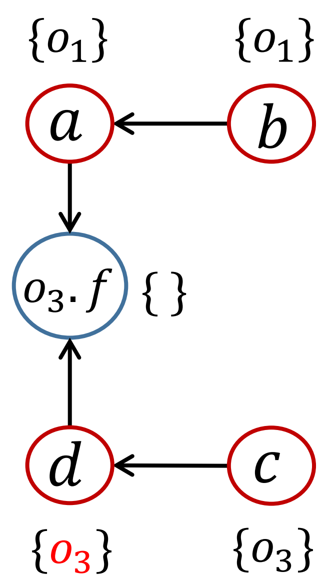
WL: $[\langle o_3.f, \{o_1\}\rangle]$

Processing: $\langle d, \{o_3\}\rangle$

What next?

PFG:



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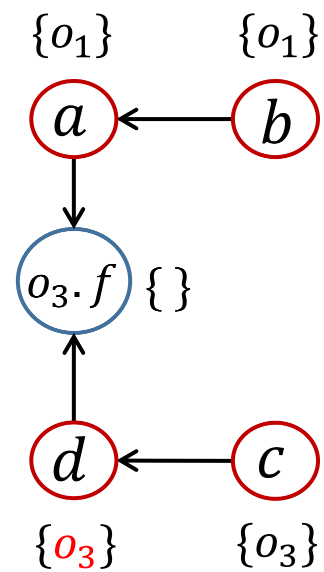
WL: $[\langle o_3.f, \{o_1\}\rangle, \langle o_3.f, \{o_3\}\rangle]$

Processing: $\langle d, \{o_3\}\rangle$

PFG:



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AddEdge(y, $o_i.f$)

foreach `y = x.f` $\in S$ **do**

AddEdge($o_i.f$, y)

Propagate(n, pts)

if pts is not empty **then**

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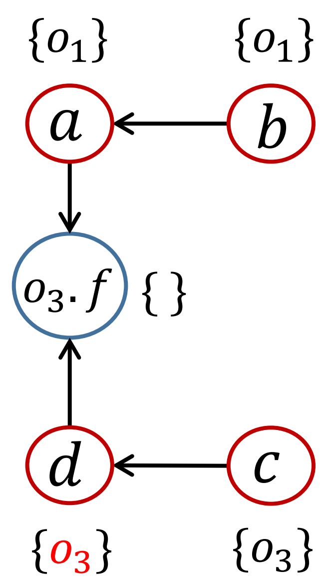
WL: $[\langle o_3.f, \{o_1\}\rangle, \langle o_3.f, \{o_3\}\rangle]$

Processing: $\langle d, \{o_3\}\rangle$

What next?

PFG:



An Example

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b = new C();
a = b;
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c.f = a;
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```

Solve(S)

$\dots$

while WL is not empty **do**

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if n represents a variable `x` **then**

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AddEdge(s, t)

if $s \rightarrow t \notin PFG$ **then**

add $s \rightarrow t$ to PFG

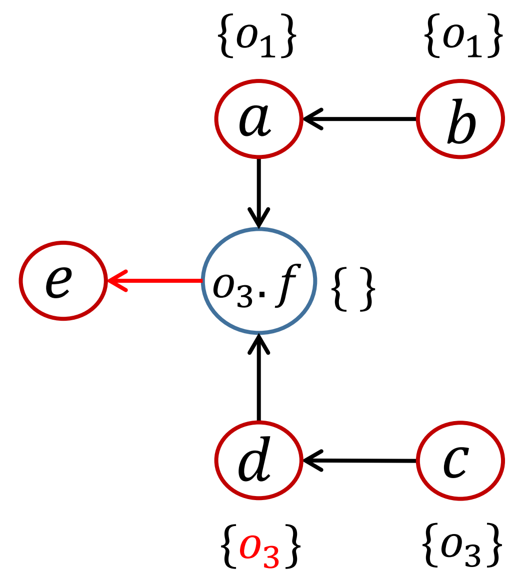
if $pt(s)$ is not empty **then**

add $\langle t, pt(s) \rangle$ to WL

WL: $[\langle o_3.f, \{o_1\} \rangle, \langle o_3.f, \{o_3\} \rangle]$

Processing: $\langle d, \{o_3\} \rangle$

PFG:



An Example

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Propagate(n, pts)
    if pts is not empty **then**
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        foreach $n \rightarrow s \in PFG$ **do**
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S:

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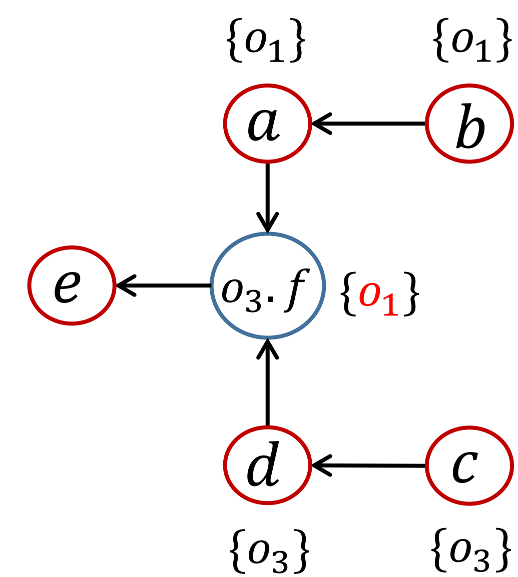
d = c;
c.f = d;
e = d.f;
```

WL: $[\langle o_3.f, \{o_3\} \rangle, \langle e, \{o_1\} \rangle]$

Processing: $\langle o_3.f, \{o_1\} \rangle$

PFG:



An Example

Solve(S)
…
while WL is not empty **do**
remove $\langle n, pts\rangle$ from WL
$\Delta = pts - pt(n)$
Propagate(n, Δ)
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AddEdge($o_i.f, y$)

Propagate(n, pts)
if pts is not empty **then**
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foreach $n \rightarrow s \in PFG$ **do**
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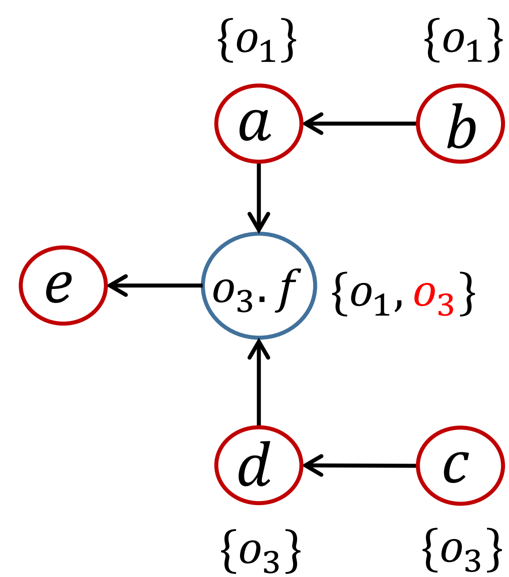
S:

```
b = new C();
a = b;
c = new C();
c.f = a;
d = c;
c.f = d;
e = d.f;
```

WL: $[\langle e, \{o_1\}\rangle, \langle e, \{o_3\}\rangle]$

Processing: $\langle o_3.f, \{o_3\}\rangle$

PFG:



An Example

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        foreach `y = x.f` $\in S$ **do**
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Propagate(n, pts)
  if pts is not empty **then**
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    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts \rangle$ to WL

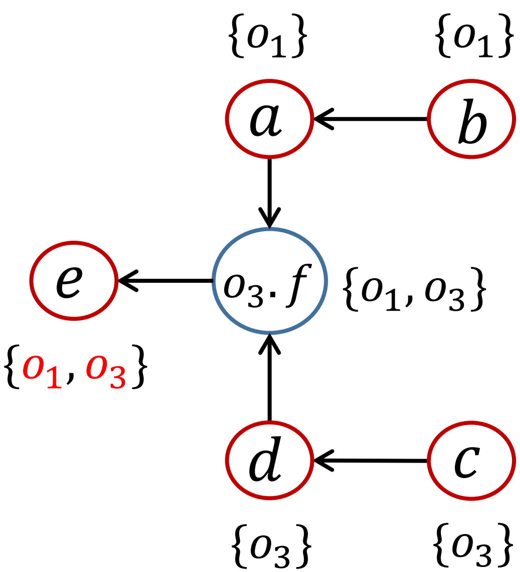
S:

```
b = new C();
a = b;
c = new C();
c.f = a;
d = c;
c.f = d;
e = d.f;
```

WL: [ ]

Processing: $\langle e, \{o_1\} \rangle, \langle e, \{o_3\} \rangle$

PFG:



An Example

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```
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a = b;
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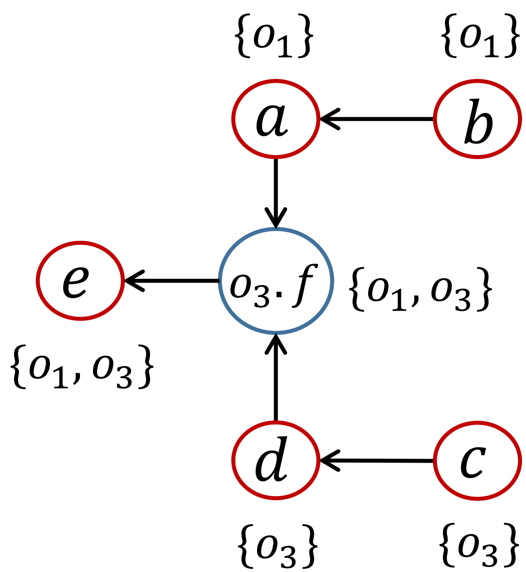
WL: [ ]

PFG:



Algorithms: Review

Solve(S)
  WL = [ ], PFG = {}
  foreach `i: x = new T()` $\in S$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL

  foreach `x = y` $\in S$ **do**
    AddEdge(y, x)

  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)

main algorithm

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then**
    add $s \rightarrow t$ to PFG
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s)\rangle$ to WL

Propagate(n, pts)
  if pts is not empty **then**
    $pt(n)$ ∪= pts
    foreach $n \rightarrow s \in PFG$ **do**
      add $\langle s, pts\rangle$ to WL

S	Set of statements of the input program
WL	Work list
PFG	Pointer flow graph

The X You Need To Understand in This Lecture

- Understand pointer analysis rules
- Understand pointer flow graph
- Understand pointer analysis algorithms

Static Program Analysis

Pointer Analysis Foundations (II)

Nanjing University

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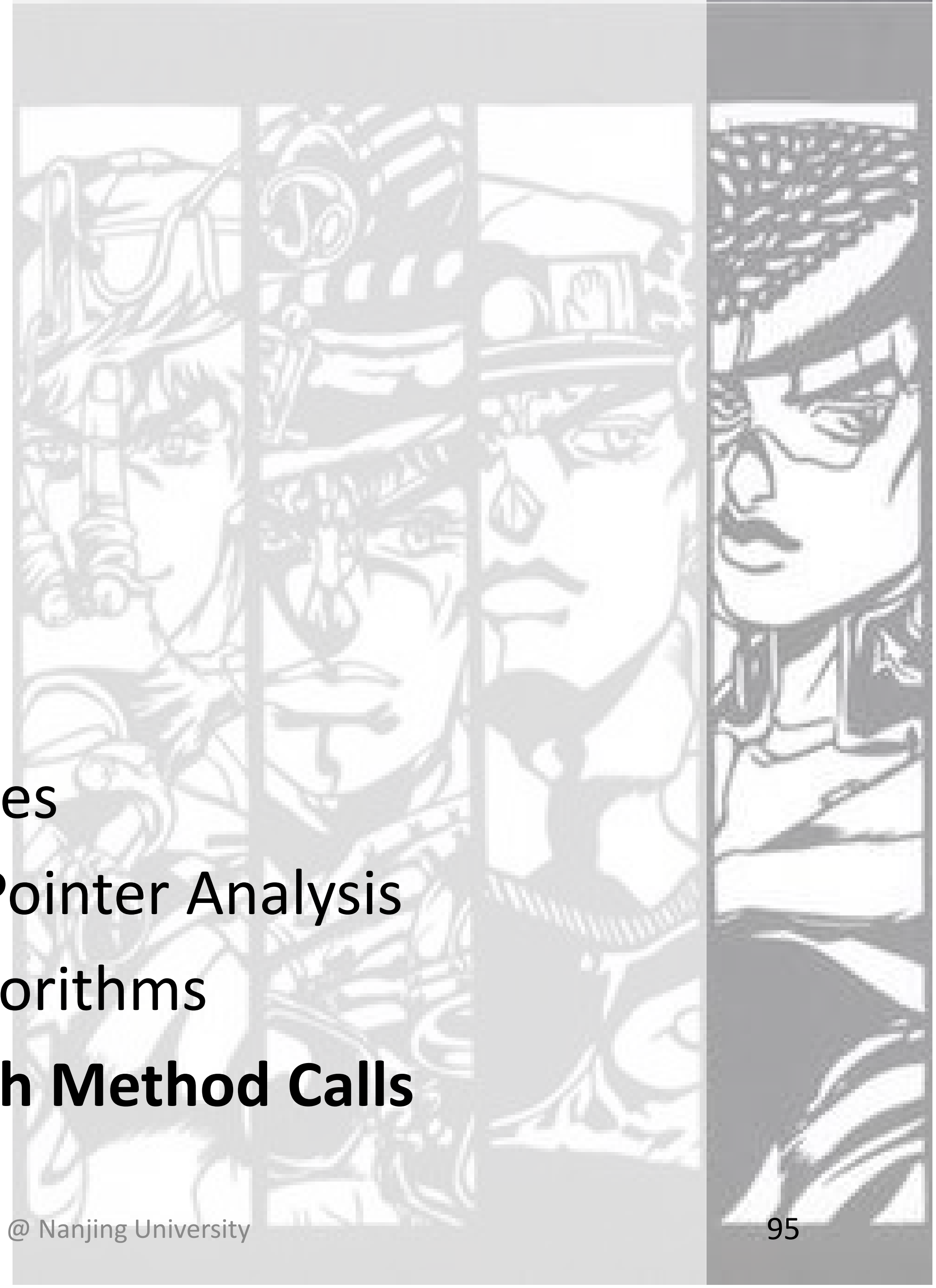
2021

Contents



Pointer Analysis in the Presence of Method Invocations

- Inter-procedural pointer analysis requires call graph

```
void foo(A a) {
    …
    b = a.bar();
    …
}
```

$pt(a) = ???$

$pt(b) = ???$

Pointer Analysis in the Presence of Method Invocations

- Inter-procedural pointer analysis requires call graph

CHA: resolve call targets based on declared type of a

```
void foo(A a) {
    …
    b = a.bar();
    …
}
```

$pt(a) = ???$

$pt(b) = ???$

- Call graph construction
  - CHA: imprecise, introduce spurious call graph edges and points-to relations

Pointer Analysis in the Presence of Method Invocations

- Inter-procedural pointer analysis requires call graph

CHA: resolve call targets based on declared type of a

Pointer analysis: resolve call targets based on $pt(a)$

```
void foo(A a) {
    …
    b = a.bar();
    …
}
```

$pt(a) = ???$

$pt(b) = ???$

- Call graph construction
  - CHA: imprecise, introduce spurious call graph edges and points-to relations
  - Pointer analysis: more precise than CHA, both for call graph and points-to relations

a.k.a on-the-fly call graph construction

Rule: Call

Kind	Statement	Rule
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$

Rule: Call

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Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ \ m = \mathrm{Dispatch}(o_i, \mathrm{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$

- $\mathrm{Dispatch}(o_i, \mathrm{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)

Rule: Call

Kind	Statement	Rule
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(\boldsymbol{x}),\ m = \text{Dispatch}(o_i, \mathrm{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(\boldsymbol{m_{this}}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$

- $\text{Dispatch}(o_i, \mathrm{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
- m_{this}: this variable of m

Rule: Call

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```
C x = new T();
…
r = x.foo(a1, a2);
```

```
class T … {    …
  B foo(A p1, A p2) {
    this…
    return ret;
  }}
```

- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
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Rule: Call

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- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
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- m_{pj}: the *j*-th parameter of m

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```

Rule: Call

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{\begin{array}{c} o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \\ o_u \in pt(\boldsymbol{aj}), 1 \le j \le n \\ o_v \in pt(m_{ret}) \end{array}}{\begin{array}{c} o_i \in pt(m_{this}) \\ o_u \in pt(\boldsymbol{m_{pj}}), 1 \le j \le n \\ o_v \in pt(r) \end{array}}$	$a1 \to m_{p1}$ … $an \to m_{pn}$

- $\text{Dispatch}(o_i, \text{k})$: resolves the virtual dispatch of k on o_i to a target method (based on type of o_i)
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- m_{pj}: the *j*-th parameter of m
- m_{ret}: the variable that holds the return value of m

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r = x.foo(a1, a2);
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class T … {  …
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    this…
    return ret;
}}
```

Rule: Call

Why not add PFG edge $x \rightarrow m_{this}$?

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

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$pt(x) = \{$ `new A`, `new B`, `new C` $\}$

```
x.foo();
```

```
class A {
  T foo() {
    this…
}}
```

```
class B extends A {
  T foo() {
    this…?
}}
```

```
class C extends A {
  T foo() {
    this…
}}
```

Rule: Call

Why not add PFG edge $x \rightarrow m_{this}$?

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$pt(x) =$ { `new A`, `new B`, `new C` }

```
x.foo();
```

```
class A {
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    this… ?
}}
```

```
class B extends A {
  T foo() {
    this…
}}
```

```
class C extends A {
  T foo() {
    this…
}}
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Rule: Call

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Receiver object should only flow to `this` variable of the **corresponding** target method

PFG edge $x \rightarrow m_{this}$ would introduce **spurious** points-to relations for `this` variables

$pt(x) = \{$ `new A`, `new B`, `new C` $\}$

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x.foo();
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Rule: Call

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Receiver object should only flow to `this` variable of the **corresponding** target method

PFG edge $x \rightarrow m_{this}$ would introduce **spurious** points-to relations for `this` variables

$pt(x) = \{$ `new A`, `new B`, `new C` $\}$

With $x \rightarrow m_{this}$

```
x.foo();
```

```
class A {
  T foo() {
    this…
}}
```

```
class B extends A {
  T foo() {
    this…
}}
```

```
class C extends A {
  T foo() {
    this…
}}
```

Rule: Call

Why not add PFG edge $x \rightarrow m_{this}$?

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$$\frac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \leq j \leq n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \leq j \leq n \quad o_v \in pt(r)}$$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

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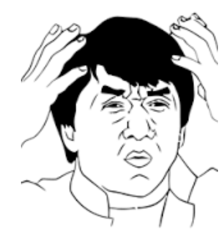
PFG edge $x \rightarrow m_{this}$ would introduce **spurious** points-to relations for `this` variables

$pt(x) = \{$ `new A`, `new B`, `new C` $\}$

```
x.foo();
```

With $x \rightarrow m_{this}$



```
class A {
  T foo() {
    this…
}}
```

{ new A, new B, new C }

```
class B extends A {
  T foo()
    this…
}}
```

{ new A, new B, new C }

```
class C extends A {
  T foo()
    this…
}}
```

{ new A, new B, new C }

Rule: Call

Why not add PFG edge $x \rightarrow m_{this}$?

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \textcolor{red}{\text{Dispatch}}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

Receiver object should only flow to `this` variable of the **corresponding** target method

PFG edge $x \rightarrow m_{this}$ would introduce **spurious** points-to relations for `this` variables

$pt(x) =$ { `new A`, `new B`, `new C` }

```
x.foo();
```

Without $x \rightarrow m_{this}$

```
class A {
  T foo() {
    this…
}}
```

{ `new A` }

```
class B extends A {
  T foo() {
    this…
}}
```

{ `new B` }

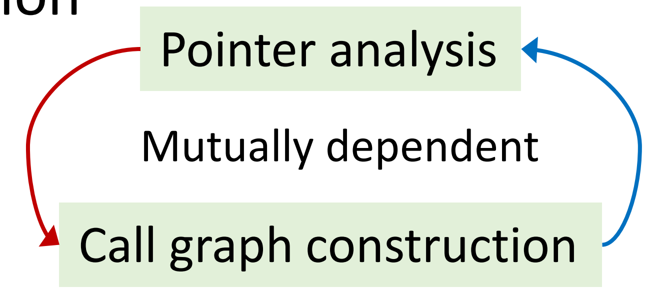
```
class C extends A {
  T foo() {
    this…
}}
```

{ `new C` }

Interprocedural Pointer Analysis

- Run together with call graph construction

```
void foo(A a) {
    …
    b = a.bar();
    …
}
```



Interprocedural Pointer Analysis

- Run together with call graph construction

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void foo(A a) {
    …
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}
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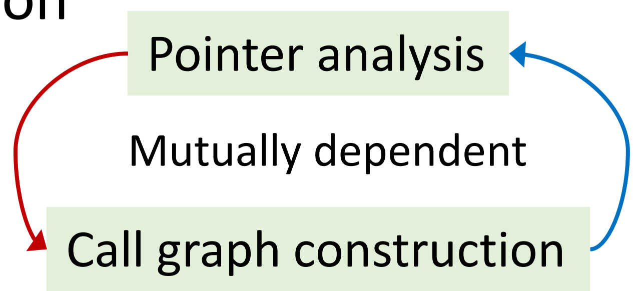
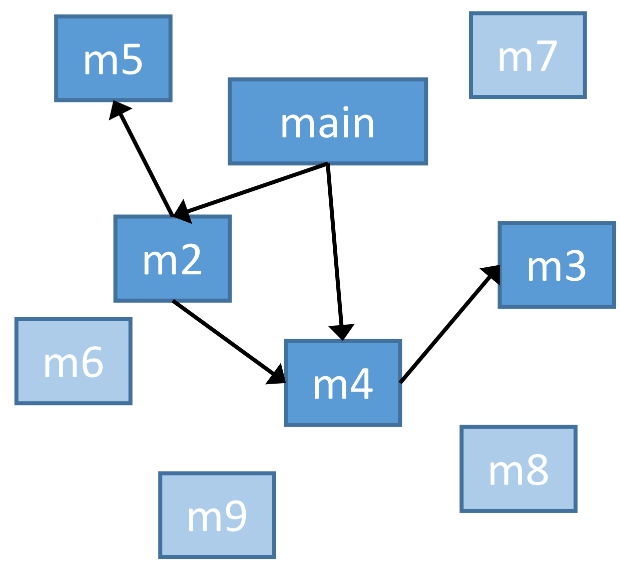


- Call graph forms a "reachable world"
  - Entry methods (e.g., the main method) are reachable from the beginning
  - The other reachable methods are gradually discovered during analysis
  - Only reachable methods and statements are analyzed



Algorithms

New part

Solve(m^{entry})
  WL=[ ],PFG={},S={},RM={},CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable x **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of reachable methods
CG	Call graph edges

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\}\rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

AddReachable(m)

- Expand the "reachable world", called
  - at the beginning for entry methods
  - when new call graph edge is discovered

AddReachable(m)
  if $m \notin RM$ **then**
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    foreach `i: x = new T()` $\in S_m$ **do**
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S_m	Set of statements in method m
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    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

Add new reachable method and statements

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of reachable methods

AddReachable(m)

- Expand the "reachable world", called
  - at the beginning for entry methods
  - when new call graph edge is discovered

AddEdge(s, t)
  if $s \rightarrow t \notin PFG$ **then**
    add $s \rightarrow t$ to PFG
    if $pt(s)$ is not empty **then**
      add $\langle t, pt(s) \rangle$ to WL

(Same as before)

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

Add new reachable method and statements

Update worklist and PFG for new discovered statements

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of reachable methods

Algorithms

New part

$\text{Solve}(m^{entry})$
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  $\text{AddReachable}(m^{entry})$
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    $\text{Propagate}(n, \Delta)$
    if n represents a variable x **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          $\text{AddEdge}(y, o_i.f)$
        foreach `y = x.f` $\in S$ **do**
          $\text{AddEdge}(o_i.f, y)$
        $\text{ProcessCall}(x, o_i)$

S	Set of reachable statements
S_m	Set of statements in method m
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$\text{AddReachable}(m)$
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      $\text{AddEdge}(y, x)$

$\text{ProcessCall}(x, o_i)$
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    $m = \text{Dispatch}(o_i, \text{k})$
    add $\langle m_{this}, \{o_i\} \rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      $\text{AddReachable}(m)$
      foreach parameter p_i of m **do**
        $\text{AddEdge}(a_i, p_i)$
      $\text{AddEdge}(m_{ret}, r)$

ProcessCall(x, o_i)

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foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
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add $\langle m_{this}, \{o_i\}\rangle$ to WL
if $l \rightarrow m \notin CG$ **then**
add $l \rightarrow m$ to CG
AddReachable(m)
foreach parameter p_i of m **do**
AddEdge(a_i, p_i)
AddEdge(m_{ret}, r)

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

ProcessCall(x, o_i)

```
ProcessCall(x, oᵢ)
    foreach l: r = x.k(a1,…,an) ∈ S do
        m = Dispatch(oᵢ, k)
        add ⟨m_this, {oᵢ}⟩ to WL
        if l → m ∉ CG then
            add l → m to CG
            AddReachable(m)
            foreach parameter pᵢ of m  do
                AddEdge(aᵢ, pᵢ)
            AddEdge(m_ret, r)
```

Pass receiver object to *this* variable

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

ProcessCall(x, o_i)

ProcessCall(x, o_i)
    foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
        m = Dispatch(o_i, k)
        add $\langle m_{this}, \{o_i\}\rangle$ to WL
        if $l \rightarrow m \notin CG$ **then**
            add $l \rightarrow m$ to CG
            AddReachable(m)
            foreach parameter p_i of m **do**
                AddEdge(a_i, p_i)
            AddEdge(m_{ret}, r)

Pass receiver object to *this* variable

Construct call graph on the fly

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

ProcessCall(x, o_i)

ProcessCall(x, o_i)
foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
m = Dispatch(o_i, k)
add $\langle m_{this}, \{o_i\}\rangle$ to WL
if $l \rightarrow m \notin CG$ **then**
add $l \rightarrow m$ to CG
AddReachable(m)
foreach parameter p_i of m **do**
AddEdge(a_i, p_i)
AddEdge(m_{ret}, r)

Pass receiver object to *this* variable

Construct call graph on the fly

Pass arguments

Pass return values

Kind	Statement	Rule	PFG Edge
Call	`l: r = x.k(a1,…,an)`	$\dfrac{o_i \in pt(x),\ m = \text{Dispatch}(o_i, \text{k}) \quad o_u \in pt(aj), 1 \le j \le n \quad o_v \in pt(m_{ret})}{o_i \in pt(m_{this}) \quad o_u \in pt(m_{pj}), 1 \le j \le n \quad o_v \in pt(r)}$	$a1 \rightarrow m_{p1}$ … $an \rightarrow m_{pn}$ $r \leftarrow m_{ret}$

Algorithms Output: Points-to Relations (pt) Call Graph (CG)

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable x **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

S	Set of reachable statements
S_m	Set of statements in method m
RM	Set of reachable methods
CG	Call graph edges

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\} \rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{o_i\} \rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

An Example

Solve(m^{entry})
WL=[ ],PFG={},S={},RM={},CG={}
AddReachable(m^{entry})
while WL is not empty **do**
remove $\langle n, pts \rangle$ from WL
$\Delta = pts - pt(n)$
Propagate(n, Δ)
if n represents a variable `x` **then**
foreach $o_i \in \Delta$ **do**
foreach `x.f = y` $\in S$ **do**
AddEdge(y, $o_i.f$)
foreach `y = x.f` $\in S$ **do**
AddEdge($o_i.f$, y)
ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

WL: [ ]

RM: { }

CG: { }

PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
AddReachable(m^{entry})

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\}\rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

WL: [ ]

RM: { }

CG: { }

PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\}\rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

```java
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

WL: [ ]

RM: { A.main() }

CG: { }

PFG:

An Example

Solve(m^{entry})

  WL=[ ], PFG={}, S={}, RM={}, CG={}

  AddReachable(m^{entry})

AddReachable(m)

  if $m \notin RM$ **then**

    add m to RM

    S ∪= S_m

    foreach `i: x = new T()` ∈ S_m **do**

      add ⟨x, {o_i}⟩ to WL

    foreach `x = y` ∈ S_m **do**

      AddEdge(y, x)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

WL: [⟨a, {o_3}⟩, ⟨b, {o_4}⟩]

RM: { A.main() }

CG: { }

PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

WL: $[\langle b, \{o_4\}\rangle]$

RM: { A.main() }

Processing:
$\langle a, \{o_3\}\rangle$

CG: { }

PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

$\{o_3\}$

a

WL: $[\langle b, \{o_4\} \rangle]$

RM: { A.main() }

Processing:
$\langle a, \{o_3\} \rangle$

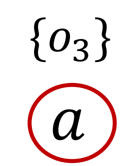
CG: { }

PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```



WL: [ ]

RM: { A.main() }

Processing:
$\langle b, \{o_4\}\rangle$

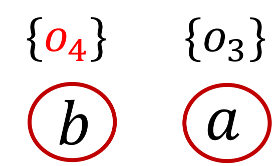
CG: { }

PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```



WL: [ ]

RM: { A.main() }

Processing:
$\langle b, \{o_4\} \rangle$

CG: { }

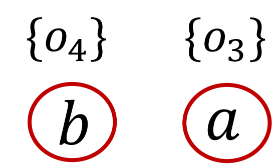
PFG:

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts\rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

What next?

```java
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```



WL: [ ]

RM: { A.main() }

Processing:
$\langle b, \{o_4\}\rangle$

CG: { }

PFG:

An Example

ProcessCall(x, o_i)
foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
  m = Dispatch(o_i, k)
  add $\langle m_{this}, \{o_i\}\rangle$ to WL
  if $l \rightarrow m \notin CG$ **then**
    add $l \rightarrow m$ to CG
    AddReachable(m)
    foreach parameter p_i of m **do**
      AddEdge(a_i, p_i)
    AddEdge(m_{ret}, r)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

ProcessCall(x, o_i)

$\{o_4\}$ b $\{o_3\}$ a

WL: [ ]

RM: { A.main() }

Processing:
$\langle b, \{o_4\}\rangle$

CG: { }

PFG:

An Example

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

$m =$ **?**

ProcessCall(x, o_i)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

$\{o_4\}$ $\{o_3\}$

b a

WL: [ ]

RM: { A.main() }

CG: { }

PFG:

Processing:
$\langle b, \{o_4\}\rangle$

An Example

$ProcessCall(x, o_i)$
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

m = B.foo(A)

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

$\{o_4\}$ $\{o_3\}$

b a

$ProcessCall(x, o_i)$

WL: [ ]

RM: { A.main() }

CG: { }

PFG:

Processing:
$\langle b, \{o_4\}\rangle$

An Example

$\text{ProcessCall}(x, o_i)$
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    $m = \text{Dispatch}(o_i, \text{k})$
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \to m \notin CG$ **then**
      add $l \to m$ to CG
      $\text{AddReachable}(m)$
      foreach parameter p_i of m **do**
        $\text{AddEdge}(a_i, p_i)$
      $\text{AddEdge}(m_{ret}, r)$

$\text{ProcessCall}(x, o_i)$

```
class A {
  static void main() {
    A a = new A();
    A b = new B();
    A c = b.foo(a);
  }
  A foo(A x) { … }
}
class B extends A {
  A foo(A y) {
    A r = new A();
    return r;
  }
}
```

$\{o_4\}$ b $\{o_3\}$ a

WL: $[\langle B.foo/this, \{o_4\}\rangle]$

RM: { A.main() }

CG: { }

PFG:

Processing:
$\langle b, \{o_4\}\rangle$

An Example

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \to m \notin CG$ **then**
      add $l \to m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
6    }
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

$\{o_4\}$ b $\{o_3\}$ a

ProcessCall(x, o_i)

WL: $[\langle B.foo/this, \{o_4\}\rangle]$

RM: { A.main() }

CG: { 5 → B.foo(A) }

PFG:

Processing:
$\langle b, \{o_4\}\rangle$

An Example

$\text{ProcessCall}(x, o_i)$

foreach `l: r = x.k(a1,…,an)` $\in S$ **do**

$m = \text{Dispatch}(o_i, \text{k})$

add $\langle m_{this}, \{o_i\}\rangle$ to WL

if $l \rightarrow m \notin CG$ **then**

add $l \rightarrow m$ to CG

$\text{AddReachable}(m)$

foreach parameter p_i of m **do**

$\text{AddEdge}(a_i, p_i)$

$\text{AddEdge}(m_{ret}, r)$

$\text{ProcessCall}(x, o_i)$

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
6    }
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

$\{o_4\}$ b $\{o_3\}$ a

WL: $[\langle \text{B.foo}/this, \{o_4\}\rangle]$

RM: { A.main() }

CG: { $5 \rightarrow$ B.foo(A) }

PFG:

Processing:

$\langle b, \{o_4\}\rangle$

CHA: { $5 \rightarrow$ B.foo(A) , $5 \rightarrow$ A.foo(A) }

Spurious call edge

An Example

ProcessCall(x, o_i)
foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
m = Dispatch(o_i, k)
add $\langle m_{this}, \{o_i\}\rangle$ to WL
if $l \rightarrow m \notin CG$ **then**
add $l \rightarrow m$ to CG
AddReachable(m)
foreach parameter p_i of m **do**
AddEdge(a_i, p_i)
AddEdge(m_{ret}, r)

What change?

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
6    }
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

$\{o_4\}$ b $\{o_3\}$ a

ProcessCall(x, o_i)

WL: $[\langle B.foo/this, \{o_4\}\rangle]$

RM: { A.main() }

CG: { $5 \rightarrow$ B.foo(A) }

PFG:

Processing:
$\langle b, \{o_4\}\rangle$

An Example

$\color{red}{\text{ProcessCall}}(x, o_i)$
foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
  $m = \color{red}{\text{Dispatch}}(o_i, \text{k})$
  add $\langle m_{this}, \{o_i\}\rangle$ to WL
  if $l \to m \notin CG$ **then**
    add $l \to m$ to CG
    $\color{red}{\text{AddReachable}}(m)$
    foreach parameter p_i of m **do**
      $\color{red}{\text{AddEdge}}(a_i, p_i)$
    $\color{red}{\text{AddEdge}}(m_{ret}, r)$

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
6    }
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

$\color{red}{\text{AddReachable}}(m)$
if $m \notin RM$ **then**
  add m to RM
  $S \cup= S_m$
  foreach `i: x = new T()` $\in S_m$ **do**
    add $\langle x, \{o_i\}\rangle$ to WL
  foreach `x = y` $\in S_m$ **do**
    $\color{red}{\text{AddEdge}}(y, x)$

$\color{red}{\text{ProcessCall}}(x, o_i)$

WL: $[\langle \text{B.foo}/this, \{o_4\}\rangle]$

RM: { A.main() }

CG: $\{ 5 \to \text{B.foo(A)} \}$

Processing:
$\langle b, \{o_4\}\rangle$

An Example

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch$(o_i$, k)
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
6    }
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

AddReachable(m)
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\}\rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      AddEdge(y, x)

ProcessCall(x, o_i)

WL: $[\langle \text{B.foo}/this, \{o_4\}\rangle]$

RM: { A.main(), B.foo(A) }

CG: $\{ 5 \rightarrow$ B.foo(A) $\}$

Processing:
$\langle b, \{o_4\}\rangle$

An Example

$\textcolor{red}{\text{ProcessCall}}(x, o_i)$
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    $m = \textcolor{red}{\text{Dispatch}}(o_i, \text{k})$
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \rightarrow m \notin CG$ **then**
      add $l \rightarrow m$ to CG
      $\textcolor{red}{\text{AddReachable}}(m)$
      foreach parameter p_i of m **do**
        $\textcolor{red}{\text{AddEdge}}(a_i, p_i)$
      $\textcolor{red}{\text{AddEdge}}(m_{ret}, r)$

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
6    }
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

$\textcolor{red}{\text{ProcessCall}}(x, o_i)$

$\textcolor{red}{\text{AddReachable}}(m)$
  if $m \notin RM$ **then**
    add m to RM
    $S \cup= S_m$
    foreach `i: x = new T()` $\in S_m$ **do**
      add $\langle x, \{o_i\}\rangle$ to WL
    foreach `x = y` $\in S_m$ **do**
      $\textcolor{red}{\text{AddEdge}}(y, x)$

WL: $[\langle B.\text{foo}/this, \{o_4\}\rangle,$ $\textcolor{red}{\langle r, \{o_{11}\}\rangle}]$

RM: { A.main(), B.foo(A) }

CG: { 5 → B.foo(A) }

Processing:
$\langle b, \{o_4\}\rangle$

An Example

ProcessCall(x, o_i)
  foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
    m = Dispatch(o_i, k)
    add $\langle m_{this}, \{o_i\}\rangle$ to WL
    if $l \to m \notin CG$ **then**
      add $l \to m$ to CG
      AddReachable(m)
      foreach parameter p_i of m **do**
        AddEdge(a_i, p_i)
      AddEdge(m_{ret}, r)

ProcessCall(x, o_i)

```
1  class A {
2    static void main() {
3      A a = new A();
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9  class B extends A {
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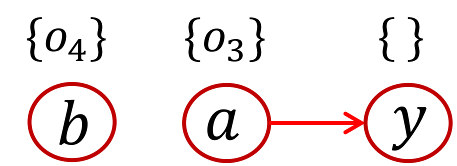
14 }
```



WL: [$\langle B.foo/this, \{o_4\}\rangle$, $\langle r, \{o_{11}\}\rangle$, $\langle y, \{o_3\}\rangle$]

RM: { A.main(), B.foo(A) }

CG: { $5 \to$ B.foo(A) }

PFG:

Processing:
$\langle b, \{o_4\}\rangle$

An Example

$\text{ProcessCall}(x, o_i)$
foreach `l: r = x.k(a1,…,an)` $\in S$ **do**
  $m = \text{Dispatch}(o_i, \text{k})$
  add $\langle m_{this}, \{o_i\}\rangle$ to WL
  if $l \rightarrow m \notin CG$ **then**
    add $l \rightarrow m$ to CG
    $\text{AddReachable}(m)$
    foreach parameter p_i of m **do**
      $\text{AddEdge}(a_i, p_i)$
    $\text{AddEdge}(m_{ret}, r)$

$\text{ProcessCall}(x, o_i)$

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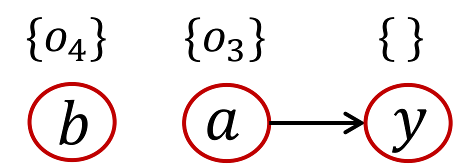
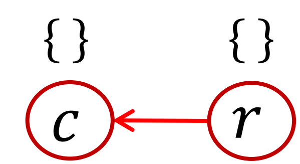


WL: $[\langle \text{B.foo}/this, \{o_4\}\rangle, \langle r, \{o_{11}\}\rangle, \langle y, \{o_3\}\rangle]$

RM: { A.main(), B.foo(A) }

CG: $\{ 5 \rightarrow \text{B.foo(A)} \}$

PFG:



Processing:
$\langle b, \{o_4\}\rangle$

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
    remove $\langle n, pts \rangle$ from WL
    $\Delta = pts - pt(n)$
    Propagate(n, Δ)
    if n represents a variable `x` **then**
      foreach $o_i \in \Delta$ **do**
        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
1  class A {
2    static void main() {
3      A a = new A();
4      A b = new B();
5      A c = b.foo(a);
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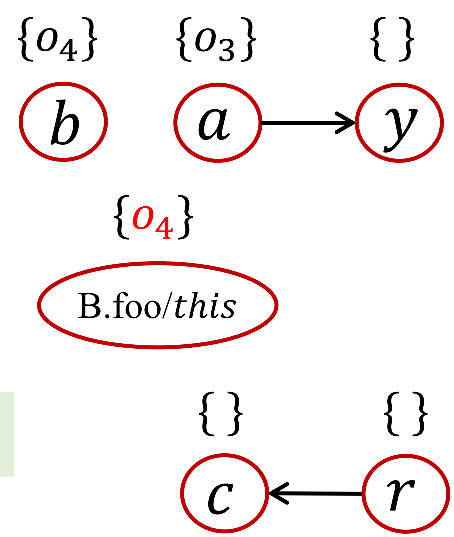
7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

WL: $[\langle r, \{o_{11}\}\rangle, \langle y, \{o_3\}\rangle]$

RM: { A.main(), B.foo(A) }

CG: { 5 → B.foo(A) }

PFG:



Processing:
$\langle \mathrm{B.foo}/this, \{o_4\} \rangle$

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
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  while WL is not empty **do**
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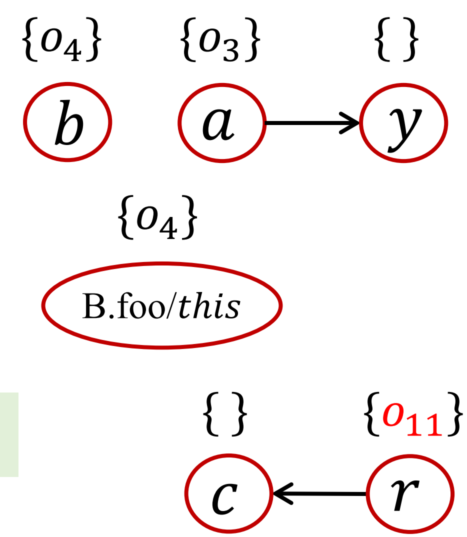

WL: $[\langle y, \{o_3\} \rangle, \langle c, \{o_{11}\} \rangle]$

RM: { A.main(), B.foo(A) }

CG: { 5 → B.foo(A) }

PFG:

Processing:
$\langle r, \{o_{11}\} \rangle$

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
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          AddEdge(y, $o_i.f$)
        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

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8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

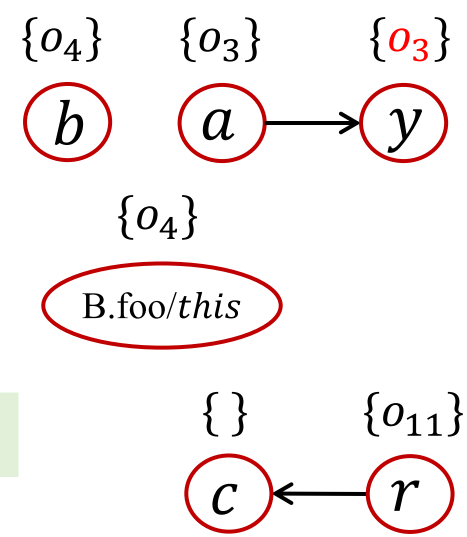
WL: $[\langle c, \{o_{11}\} \rangle]$

RM: { A.main(), B.foo(A) }

CG: { 5 → B.foo(A) }

Processing:
$\langle y, \{o_3\} \rangle$

PFG:



An Example

Solve(m^{entry})
  WL=[ ],PFG={},S={},RM={},CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
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        foreach `y = x.f` $\in S$ **do**
          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
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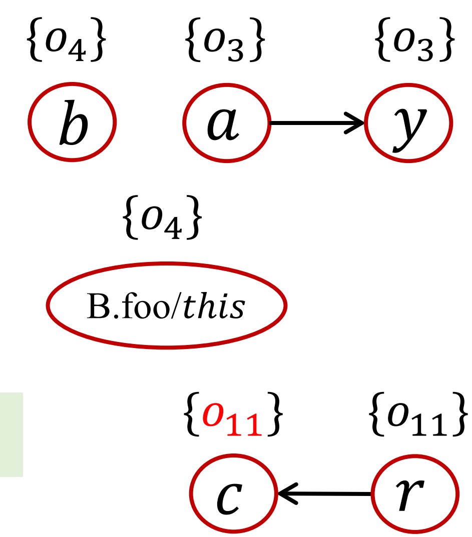
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

WL: [ ]

RM: { A.main(), B.foo(A) }

CG: { 5 → B.foo(A) }

PFG:



Processing:
  $\langle c, \{o_{11}\} \rangle$

An Example

Solve(m^{entry})
  WL=[ ], PFG={}, S={}, RM={}, CG={}
  AddReachable(m^{entry})
  while WL is not empty **do**
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    $\Delta = pts - pt(n)$
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        foreach `x.f = y` $\in S$ **do**
          AddEdge(y, $o_i.f$)
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          AddEdge($o_i.f$, y)
        ProcessCall(x, o_i)

```
1  class A {
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3      A a = new A();
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7    A foo(A x) { … }
8  }
9  class B extends A {
10   A foo(A y) {
11     A r = new A();
12     return r;
13   }
14 }
```

Algorithm finishes

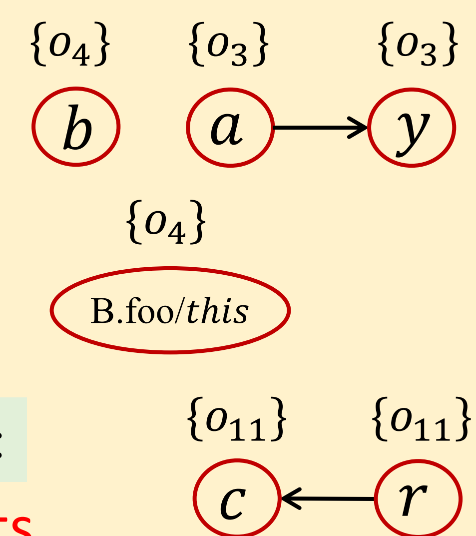
WL: [ ]

RM: { A.main(), B.foo(A) }

Processing:

CG: { 5 → B.foo(A) }

PFG:



Final results

The X You Need To Understand in This Lecture

- Understand pointer analysis rule for method call
- Understand inter-procedural pointer analysis algorithm
- Understand on-the-fly call graph construction